

3.2 Find integrals and distinguish signed accumulation from area

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS indefinite integral 不定积分 • definite integral 定积分
• constant of integration 积分常数

Objective

Find integrals and distinguish signed accumulation from area. Original GAC004-aligned practice, not an official assessment. Show the method as well as the result. All applied scenarios are invented classroom problems.

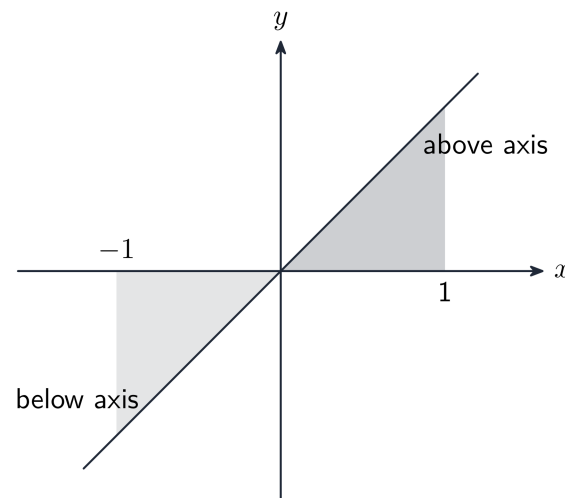
1 Key knowledge and worked examples

■ **Key knowledge: a definite integral can cancel contributions with opposite signs**

An **indefinite integral** 不定积分 gives a family of antiderivatives with a **constant of integration** 积分常数 C . The power rule is $\int x^n dx = x^{n+1}/(n+1) + C$ for $n \neq -1$, on a suitable domain. For the reciprocal, $\int 1/x dx = \ln|x| + C$ on an interval not crossing zero.

A **definite integral** 定积分 is signed accumulation. When an antiderivative F is valid over the interval, $\int_a^b f(x) dx = F(b) - F(a)$. Geometric area is nonnegative, so split where a curve changes sign. Substitution reverses the chain rule and requires matching differential factors; definite-integral bounds must change consistently or return to the original variable.

■ **Worked example: negative and positive areas can cancel**



Known: $f(x) = x$ from negative 1 to 1. Its definite integral is zero because the negative and positive contributions cancel.

$$I = \int_{-1}^1 x dx = \left[\frac{x^2}{2} \right]_{-1}^1 = \frac{1}{2} - \frac{1}{2} = 0$$

Geometric area instead adds the magnitudes of the two triangular regions.

$$A = - \int_{-1}^0 x dx + \int_0^1 x dx = \frac{1}{2} + \frac{1}{2} = 1$$

2 Practice

2.1 Find $\int (3x^2 - 4) dx$.

[2]

2.2 Evaluate $\int_0^2 (2x + 1) \, dx$. [2]

2.3 Find $\int 6x/(3x^2 + 2) \, dx$ using substitution. [3]

2.4 Find both the signed integral $\int_{-1}^1 x \, dx$ and the geometric area between that line and the x-axis over the same interval. [3]

3 Exam-style questions

3.1 A particle has velocity $v(t) = 4 - t$ metres per second for $0 \leq t \leq 6$ seconds and starts at position 5 m. Find (a) its displacement, (b) its total distance travelled, and (c) its final position. [6]

[8]

Solutions

2.1

Use the power rule term by term.

$$F(x) = x^3 - 4x + C$$

Correct antiderivative including C.

2.2

Use antiderivative $F(x) = x^2 + x$ and subtract the lower-bound value.

$$I = [x^2 + x]_0^2 = (2^2 + 2) - (0^2 + 0) = 6$$

Value 6.

2.3

Let $u = 3x^2 + 2$, so $du = 6x \, dx$.

$$I = \int \frac{1}{u} du = \ln |u| + C$$

Logarithmic antiderivative. Since $3x^2 + 2 > 0$ for real x , $I = \ln(3x^2 + 2) + C$.

2.4

The signed integral is $[x^2/2]_{-1}^1 = 0$. Split the geometric area at zero and reverse the sign of the lower region.

$$A = -\int_{-1}^0 x \, dx + \int_0^1 x \, dx = 1$$

Area 1.

3.1

(a) Integrate velocity for signed displacement.

$$\Delta s = \int_0^6 v(t) dt = \left[4t - \frac{t^2}{2} \right]_0^6 = 6 \text{ m}$$

Displacement 6 m.

(b) Velocity changes sign at $t = 4$ s. Split there and add magnitudes.

$$D = \int_0^4 (4 - t) dt - \int_4^6 (4 - t) dt = 8 + 2 = 10 \text{ m}$$

Distance 10 m.

(c) Final position $s_f = s_0 + \Delta s = 5 + 6 = 11 \text{ m}$.

3.2

(a) Equate the curve values.

$$x = x^2 - 2x \implies x(x - 3) = 0$$

Intersections at x equal to 0 and 3.

(b) The difference is $x - (x^2 - 2x) = x(3 - x) > 0$ for $0 < x < 3$, so the line is upper.

(c) Integrate upper minus lower.

$$A = \int_0^3 [x - (x^2 - 2x)] dx = \int_0^3 (3x - x^2) dx$$

$$A = \left[\frac{3}{2}x^2 - \frac{1}{3}x^3 \right]_0^3 = \frac{9}{2}$$

Correct antiderivative. Area 9/2 square units.

(d) Each half has area $\int_0^{3/2} (3x - x^2) dx = 9/4$ and $\int_{3/2}^3 (3x - x^2) dx = 9/4$. Correct half integrals. Their sum is 9/2, matching the result.