

2.8 Standardise values and interpret a stated normal model

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS normal distribution 正态分布 • z-score 标准分 • standard deviation 标准差

Objective

Standardise values and interpret a stated normal model. Original GAC004-aligned practice, not an official assessment. Show the method as well as the result. All applied scenarios are invented classroom problems.

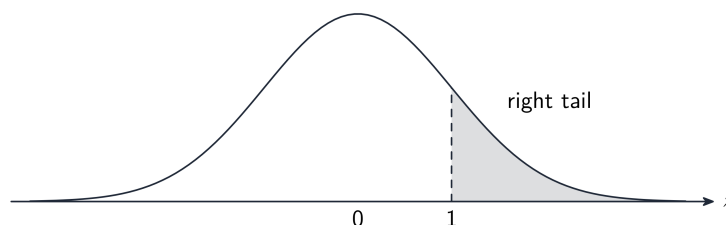
1 Key knowledge and worked examples

■ Key knowledge: normal probabilities depend on the model and its parameters

A **normal distribution** 正态分布 is a continuous symmetric bell-shaped model with mean μ and positive **standard deviation** 标准差 σ . Standardising gives a **z-score** 标准分, $z = (x - \mu)/\sigma$. Total area is 1; individual exact values have zero probability in this continuous model.

Use the supplied standard-normal cumulative values $\Phi(z) = P(Z \leq z)$: $\Phi(0) = 0.5000$, $\Phi(1) = 0.8413$, $\Phi(1.5) = 0.9332$, $\Phi(2) = 0.9772$. Symmetry gives $\Phi(-z) = 1 - \Phi(z)$. Interval probability is upper cumulative area minus lower cumulative area. These probabilities require a stated normal assumption; a mean and standard deviation alone do not prove that real data are normal.

■ Worked example: a threshold is an area beyond a standardised value



Known: an invented measurement model has $\mu = 50$ and $\sigma = 10$. For values above 60, standardise the threshold.

$$z = \frac{x - \mu}{\sigma} = \frac{60 - 50}{10} = 1$$

Use the complement of the supplied cumulative area.

$$P(X > 60) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587$$

This is a model probability, not a guarantee that exactly 15.87% of a finite sample exceeds 60.

2 Practice

2.1 A normal model has mean 100 and standard deviation 15. Find the z-score of 130.[2]

2.2 Using the supplied cumulative values, find the probability that a standard-normal value is greater than 1. [2]

2.3 Use symmetry and the supplied values to find the probability that a standard-normal value lies between negative 1 and 1. [3]

2.4 A normal model has mean 70 and standard deviation 8. Find the raw values corresponding to z-scores negative 1 and 1.5. Give the probability above the latter value. [3]

3 Exam-style questions

3.1 A stated normal model for completion time has mean 40 minutes and standard deviation 5 minutes. (a) Find the probability of completion between 35 and 50 minutes. (b) For 200 independently modelled tasks, find the expected number exceeding 50 minutes. Explain why that expectation is not a guaranteed observed count. [6]

[8]

Solutions

2.1

Standardise using the model mean and standard deviation.

$$z = \frac{x - \mu}{\sigma} = \frac{130 - 100}{15} = 2$$

Z-score 2.

2.2

Use the complement of the area at or below 1.

$$P(Z > 1) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587$$

Probability 0.1587.

2.3

Symmetry gives $\Phi(-1) = 1 - 0.8413 = 0.1587$. Subtract cumulative areas.

$$P(-1 < Z < 1) = \Phi(1) - \Phi(-1) = 0.8413 - 0.1587 = 0.6826$$

Probability 0.6826.

2.4

Invert standardisation: $x = \mu + z\sigma$.

$$x_1 = 70 + (-1)(8) = 62$$

Lower raw value.

$$x_2 = 70 + 1.5(8) = 82$$

Upper raw value.

$$P(X > 82) = 1 - \Phi(1.5) = 1 - 0.9332 = 0.0668$$

Tail probability.

3.1

(a) Standardise both bounds.

$$z_L = \frac{x_L - \mu}{\sigma} = \frac{35 - 40}{5} = -1$$

$$z_U = \frac{x_U - \mu}{\sigma} = \frac{50 - 40}{5} = 2$$

Both bounds correct. Subtract cumulative areas.

$$p = \Phi(2) - \Phi(-1) = 0.9772 - 0.1587 = 0.8185$$

Interval probability.

(b) Tail probability is $1 - 0.9772 = 0.0228$, so expected count $E = Np = 200(0.0228) = 4.56$. The random observed count is an integer and can vary; 4.56 is an expectation over repeated modelled samples.

3.2

(a) Standardise relative to each model.

$$z_A = \frac{80 - 60}{10} = 2$$

A z-score.

$$z_B = \frac{85 - 75}{5} = 2$$

B z-score. Both scores are two model standard deviations above their respective means.

(b) For each model, $p = 1 - \Phi(2) = 1 - 0.9772 = 0.0228$.

(c) Expected count on each scale is $E = Np = 500(0.0228) = 11.4$.

(d) Normality and representative parameters must be justified for the actual data. Equal standardised relative position does not establish equivalent test content, difficulty, reliability or learner ability; separate evidence is needed.