

2.7 Calculate spread with an explicit quartile and variance convention

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS range 全距 • interquartile range (IQR) 四分位距 • standard deviation 标准差

Objective

Calculate spread with an explicit quartile and variance convention. Original GAC004-aligned practice, not an official assessment. Show the method as well as the result. All applied scenarios are invented classroom problems.

1 Key knowledge and worked examples

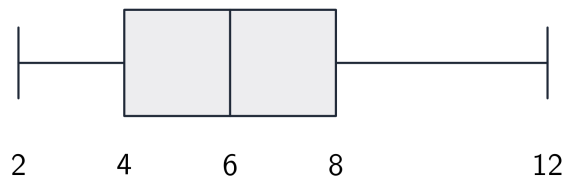
■ Key knowledge: equal means can conceal different distributions

The **range** 全距 is maximum minus minimum. The **interquartile range (IQR)** 四分位距 is $Q_3 - Q_1$. For this sheet, order the data, split an even-sized list into equal halves, and use each half's median for its quartile. For an odd list, exclude the overall median before finding half-list medians. Other software may use different conventions.

A box plot shows the median and quartiles; this sheet's whiskers run to minimum and maximum, not to an automatic outlier fence. **Standard deviation** 标准差 measures spread using squared deviations. Use population $\sigma = \sqrt{\sum(x - \mu)^2/n}$ for a complete supplied population; sample estimate $s = \sqrt{\sum(x - \bar{x})^2/(n - 1)}$ requires the stated sample convention. Standard deviation has the original units; variance has squared units.

■ Worked example: read a five-number summary rather than every observation

whiskers: minimum to maximum



Known five-number summary: minimum 2, $Q_1 = 4$, median 6, $Q_3 = 8$, maximum 12. The overall span and middle-half span are different.

$$R = x_{\max} - x_{\min} = 12 - 2 = 10$$

$$IQR = Q_3 - Q_1 = 8 - 4 = 4$$

The box does not identify every original value, the sample size or why the spread arose.

2 Practice

2.1 Find the range of 3, 5, 8 and 12. [2]

2.2 A five-number summary is minimum 4, first quartile 7, median 9, third quartile 13, maximum 20. Find its IQR and range. [2]

2.3 Using the stated half-list convention, find the median, first quartile and third quartile of 2, 4, 6, 8, 10, 12, 14, 16. [3]

[3]

[6]

8]

Solutions

2.1

Subtract the minimum from the maximum.

$$R = x_{max} - x_{min} = 12 - 3 = 9$$

Range 9.

2.2

$IQR = Q_3 - Q_1 = 13 - 7 = 6$. Range $R = 20 - 4 = 16$.

2.3

Median is the mean of the fourth and fifth values: $m = (8 + 10)/2 = 9$. Lower-half median $Q_1 = (4 + 6)/2 = 5$. Upper-half median $Q_3 = (12 + 14)/2 = 13$.

2.4

Mean $\mu = (2 + 4 + 6)/3 = 4$.

$$\sigma^2 = \frac{\sum (x - \mu)^2}{n} = \frac{(2 - 4)^2 + (4 - 4)^2 + (6 - 4)^2}{3} = \frac{8}{3}$$

Population variance.

$$\sigma = \sqrt{\sigma^2} = \sqrt{8/3} \approx 1.63$$

Population standard deviation.

3.1

Both means are 5. Use population division by n, not n minus 1.

$$\sigma_A = \sqrt{\frac{(4 - 5)^2 + (5 - 5)^2 + (6 - 5)^2}{3}} = \sqrt{2/3} \approx 0.816$$

A standard deviation.

$$\sigma_B = \sqrt{\frac{(1 - 5)^2 + (5 - 5)^2 + (9 - 5)^2}{3}} = \sqrt{32/3} \approx 3.27$$

B standard deviation. B is more spread around the shared mean. The summaries do not identify why the populations differ.

3.2

(a) Five-number summary is 2, 5, 9, 13, 16. Correct quartiles and median earn B1; a box from 5 to 13 with median 9 earns B1; whiskers to 2 and 16 on a labelled scale earn B1.

(b) Range $R = 16 - 2 = 14$. IQR $= 13 - 5 = 8$.

(c) New range $R' = 100 - 2 = 98$. Quartiles stay 5 and 13, so IQR remains 8. The range uses the changed extreme directly, while the middle-half endpoints stay fixed; neither the plot nor these summaries establishes the cause of 100. Do not delete that value without checking its origin.