

2.4 Use probability rules, conditional information and counting

Name: _____ Class: _____ Date: _____ Total: 24 marks

KEY WORDS sample space 样本空间 • independent 独立 • conditional probability 条件概率

Objective

Use probability rules, conditional information and counting. Original GAC004-aligned practice, not an official assessment. Show the method as well as the result. All applied scenarios are invented classroom problems.

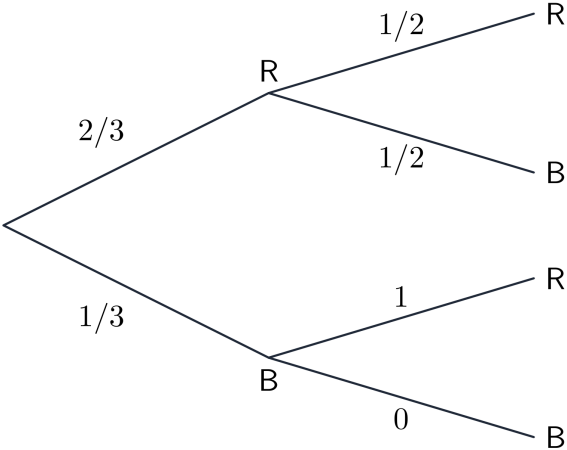
1 Key knowledge and worked examples

■ Key knowledge: independence and mutual exclusion describe different relationships

A **sample space** 样本空间 lists possible outcomes. For equally likely outcomes, probability is favourable count divided by total count. The complement has probability $1 - P(A)$. For any events, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Events are **independent** 独立 when one does not change the probability of the other; then $P(A \cap B) = P(A)P(B)$. Mutually exclusive events cannot occur together. **Conditional probability** 条件概率 uses $P(A | B) = P(A \cap B) / P(B)$ for $P(B) > 0$. Sampling without replacement changes later probabilities. Ordered arrangements use permutations; unordered selections use combinations.

■ Worked example: a second draw depends on the first



Known: two red and one blue counter, two random draws without replacement. To get two reds, follow the red-red path and multiply its conditional probabilities.

$$P(RR) = P(R_1)P(R_2 | R_1)$$

$$P(RR) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

A first blue leaves only red counters; the blue-blue path has probability zero. The draws are not independent.

2 Practice

2.1 A fair six-sided die is rolled once. Find the probability of a number greater than 4.[2]

2.2 If $P(A) = 0.35$, find the probability of the complement of A. [2]

2.3 Events have $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$. Find $P(A \cup B)$ and decide whether they are independent. [3]

2.4 Five distinct students stand in a line. (a) Count the possible orders. (b) Count the possible unordered teams of two drawn from them. [3]

3 Exam-style questions

3.1 A bag has 3 red and 2 blue counters. Two are drawn randomly without replacement. Find (a) the probability both are red and (b) the probability exactly one is red. Explain why the second draw cannot use the original proportions unchanged. [6]

[8]

Solutions

2.1

Favourable outcomes are 5 and 6 among six equally likely outcomes.

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

Probability one third.

2.2

Complement probabilities sum to 1.

$$P(A^c) = 1 - P(A) = 1 - 0.35 = 0.65$$

Probability 0.65.

2.3

Subtract the double-counted overlap.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.2 = 0.7$$

Union probability 0.7. They are independent because $P(A)P(B) = 0.4(0.5) = 0.2 = P(A \cap B)$.

2.4

Order matters for a line.

$$N_{line} = 5! = 5(4)(3)(2)(1) = 120$$

120 orders. For an unordered pair, divide ordered choices by two:

$$N_{team} = \binom{5}{2} = \frac{5(4)}{2(1)} = 10$$

10 teams.

3.1

(a) The first red leaves two red among four counters.

$$P(RR) = P(R_1)P(R_2 | R_1) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$$

Both-red probability.

(b) Exactly one red has disjoint paths RB and BR.

$$P(RB) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$$

$$P(BR) = \frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$$

Correct path products.

$$P(\text{exactly one red}) = P(RB) + P(BR) = \frac{3}{10} + \frac{3}{10} = \frac{3}{5}$$

Final probability. The remaining counts depend on which colour was removed, so the draws are not independent.

3.2

(a) Add late counts and divide by the class size.

$$P(L) = \frac{6+2}{40} = 0.20$$

Late probability.

(b) Restrict the denominator to bus users.

$$P(L | B) = \frac{6}{24} = 0.25$$

Conditional probability.

(c) Restrict the denominator to late students.

$$P(B | L) = \frac{6}{6+2} = 0.75$$

Conditional probability.

(d) Compare $P(L | B)$ with $P(L)$, or compare the joint probability with the product. They differ, so the events are not independent in the supplied table. This observational one-day table does not control other factors or randomly assign transport; association alone does not establish causation.