

2.1 Distinguish sequences from series and select a valid sum

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS arithmetic sequence 等差数列 • geometric sequence 等比数列 • series 级数

Objective

Distinguish sequences from series and select a valid sum. Original GAC004-aligned practice, not an official assessment. Show the method as well as the result. All applied scenarios are invented classroom problems.

1 Key knowledge and worked examples

■ **Key knowledge: constant differences and constant ratios give different models**

An **arithmetic sequence** 等差数列 has constant difference d , with $u_n = a + (n - 1)d$. A **geometric sequence** 等比数列 has constant ratio r , with $u_n = ar^{n-1}$. Here a is the first term and n starts at 1. A **series** 级数 adds terms of a sequence.

For n arithmetic terms, $S_n = n[2a + (n - 1)d]/2$. For geometric terms with $r \neq 1$, $S_n = a(1 - r^n)/(1 - r)$. If $r = 1$, the sum is na . For a nonzero first term, an infinite geometric sum is $S_\infty = a/(1 - r)$ only when $|r| < 1$; a finite sum can still exist when the infinite series does not converge.

■ **Worked example: row counts differ by a constant amount**

row 1: 4 • • • •

row 2: 7 • • • • • • •

row 3: 10 • • • • • • • • • •

Known: row counts are 4, 7 and 10, continuing with the same difference. Thus $a = 4$ and $d = 3$. For the fifth row:

$$u_n = a + (n - 1)d$$

$$u_5 = 4 + (5 - 1)(3) = 16$$

For the first five rows together, use the arithmetic sum, not the fifth term.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_5 = \frac{5}{2}[2(4) + (5 - 1)(3)] = 50$$

2 Practice

2.1 An arithmetic sequence begins 5, 9, 13. Find its tenth term. [2]

2.2 A geometric sequence begins 3, 6, 12. Find its sixth term. [2]

2.3 Find the sum of the first 20 terms of the arithmetic sequence with first term 7 and common difference 3. [3]

2.4 Find the sum of the first five terms of the geometric sequence with $a = 16$ and $r = 1/2$. State its infinite sum. [3]

3 Exam-style questions

3.1 An auditorium has 12 seats in its first row and 2 more seats in each subsequent row. (a) Find the seat count in row 15. (b) Find the total in the first 15 rows. (c) Find the smallest number of complete rows whose combined count is at least 400. [6]

3.2 A modelled sequence of daily deliveries is 80, 60, 45, continuing geometrically. Fractional values are allowed in this mathematical model. (a) Find its ratio and fifth term. (b) Find the sum of the first five terms. (c) Find the infinite sum and explain why it exceeds the five-term sum. (d) A different sequence is 10, 12, 14.4, continuing geometrically. Explain whether it has a finite infinite sum. [8]

Solutions

2.1

The constant difference is 4. Use the nth-term formula.

$$u_n = a + (n - 1)d$$

$$u_{10} = 5 + (10 - 1)(4) = 41$$

Tenth term 41.

2.2

The constant ratio is 2.

$$u_n = ar^{n-1}$$

$$u_6 = 3(2)^{6-1} = 96$$

Sixth term 96.

2.3

A sum includes every term, not only the last.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{20} = \frac{20}{2}[2(7) + (20 - 1)(3)]$$

Correct substitution.

$$S_{20} = 10(14 + 57) = 710$$

Sum 710.

2.4

Use the finite geometric sum.

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_5 = \frac{16(1 - (1/2)^5)}{1 - 1/2} = 31$$

Finite sum 31. Since $|r| < 1$, $S_\infty = a/(1 - r) = 16/(1/2) = 32$.

3.1

(a) Row counts form an arithmetic sequence.

$$u_n = a + (n - 1)d$$

$$u_{15} = 12 + (15 - 1)(2) = 40$$

Row 15 contains 40 seats.

(b) Add row counts.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{15} = \frac{15}{2}[24 + 28] = 390$$

Total 390.

(c) Express the total as a function of n.

$$S_n = \frac{n}{2}[24 + 2(n - 1)] = n(n + 11)$$

Check consecutive complete-row totals.

$$S_{15} = 15(26) = 390$$

$$S_{16} = 16(27) = 432$$

The totals increase for positive n, so the smallest count is 16 rows.

3.2

(a) Ratio is $r = u_2/u_1 = 60/80 = 0.75$.

$$u_n = ar^{n-1}$$

$$u_5 = 80(0.75)^4 = 25.3125$$

Fifth term.

(b) Use the geometric sum.

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_5 = \frac{80(1 - 0.75^5)}{1 - 0.75} = 244.0625$$

Five-term sum.

(c) The ratio satisfies $|r| < 1$.

$$S_\infty = \frac{a}{1 - r} = \frac{80}{1 - 0.75} = 320$$

Infinite sum 320. The remaining terms are positive, so adding them increases the sum.

(d) The second ratio is 1.2, so its positive terms grow and $|r| < 1$ fails; there is no finite infinite sum. An infinite-horizon model is not a promise about actual deliveries.