

1.4 Use similarity, right triangles, area and volume

Name: _____ Class: _____ Date: _____ Total: 24 marks

KEY WORDS similar 相似 • hypotenuse 斜边 • volume 体积

Objective

Use similarity, right triangles, area and volume. Original GAC004-aligned practice, not an official assessment. Show the method as well as the result. All applied scenarios are invented classroom problems.

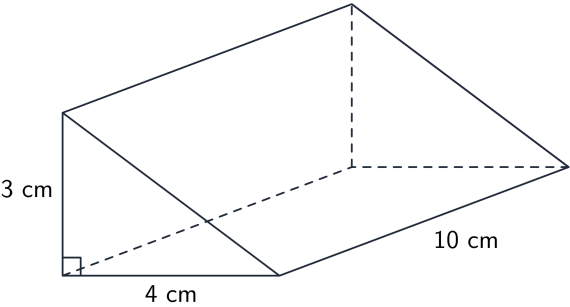
1 Key knowledge and worked examples

■ Key knowledge: choose the correct dimension and distinguish surface from volume

Similar 相似 shapes have equal corresponding angles and proportional corresponding lengths. A length scale factor k produces area factor k^2 and volume factor k^3 for similar solids. Congruent shapes have the same size as well as shape.

In a right triangle, the **hypotenuse** 斜边 is opposite the right angle and $c^2 = a^2 + b^2$. A triangle's area uses perpendicular height, not a sloping side. **Volume** 体积 measures three-dimensional space in cubic units; surface area measures the boundary in square units. For a prism, volume is cross-sectional area times length. Keep all lengths in compatible units before calculating.

■ Worked example: a triangular prism has a constant cross-section



not to scale

Known: perpendicular triangle sides are 3 and 4 cm; prism length is 10 cm. Find the cross-sectional area, then multiply by the prism length.

$$A = \frac{1}{2}bh = \frac{1}{2}(3)(4) = 6 \text{ cm}^2$$

$$V = AL = 6(10) = 60 \text{ cm}^3$$

The sloping triangle side is not its perpendicular height. Volume is not the area of the triangular end.

2 Practice

2.1 A right triangle has perpendicular sides 5 and 12 cm. Find its hypotenuse. [2]

2.2 A triangle has base 8 cm and perpendicular height 6 cm. Find its area. [2]

2.3 Two similar triangles have corresponding lengths 4 and 10 cm. The smaller has area 12 cm². Find the length scale factor and the larger area. [3]

2.4 A closed cylinder has radius 3 cm and height 8 cm. Find its volume, giving an exact answer in terms of pi and a decimal to 3 significant figures. [3]

3 Exam-style questions

3.1 A rectangular block measures 8 by 5 by 3 cm. (a) Find its volume and total surface area. (b) A geometrically similar block has all lengths doubled. Find its volume and explain why the volume is not merely doubled. [6]

3.2 An open rectangular tank has internal length 80 cm, width 50 cm and depth 40 cm. (a) Find its capacity in litres, using $1000 \text{ cm}^3 = 1 \text{ litre}$. (b) It initially contains water to depth 25 cm. Find the water volume. (c) A rectangular solid block measuring 20 by 10 by 10 cm is fully submerged without spilling; it does not absorb water. Find the new water depth and state the remaining depth below the rim. [8]

Solutions

2.1

Use Pythagoras because the given sides meet at a right angle.

$$c = \sqrt{a^2 + b^2} = \sqrt{5^2 + 12^2} = 13 \text{ cm}$$

Hypotenuse 13 cm.

2.2

Use half the rectangle area.

$$A = \frac{1}{2}bh = \frac{1}{2}(8)(6) = 24 \text{ cm}^2$$

Area 24 square centimetres.

2.3

The larger-to-smaller length factor is $k = 10/4 = 2.5$. Areas scale by the square of the length factor.

$$A_L = k^2 A_S = (2.5)^2(12) = 75 \text{ cm}^2$$

Larger area 75 square centimetres.

2.4

Use circular area times height.

$$V = \pi r^2 h = \pi(3)^2(8) = 72\pi \text{ cm}^3$$

Exact answer.

$$V = 72\pi \text{ cm}^3 \approx 226 \text{ cm}^3$$

Correct 3 significant figures.

3.1

Volume uses all three perpendicular lengths.

$$V = lwh = 8(5)(3) = 120 \text{ cm}^3$$

Volume. Sum the three pairs of opposite faces.

$$S = 2(lw + lh + wh)$$

$$S = 2(8(5) + 8(3) + 5(3)) = 158 \text{ cm}^2$$

Surface area.

$$V_L = k^3 V_S = 2^3(120) = 960 \text{ cm}^3$$

Larger volume. Each of the three dimensions doubles, so the product grows eightfold.

3.2

Capacity is internal rectangular volume.

$$V_T = lwh = 80(50)(40) = 160000 \text{ cm}^3$$

$$C = \frac{V_T}{1000} = \frac{160000}{1000} = 160 \text{ litres}$$

Capacity 160 litres.

$$V_W = lwh_W = 80(50)(25) = 100000 \text{ cm}^3 = 100 \text{ litres}$$

Initial water volume 100 litres. The submerged block displaces its own volume.

$$V_B = l_B w_B h_B = 20(10)(10) = 2000 \text{ cm}^3$$

Block volume.

$$\Delta h = \frac{V_B}{lw} = \frac{2000}{80(50)} = 0.5 \text{ cm}$$

Correct rise.

$$h_{new} = h_W + \Delta h = 25 + 0.5 = 25.5 \text{ cm}$$

New depth.

$$h_{gap} = h_T - h_{new} = 40 - 25.5 = 14.5 \text{ cm}$$

Remaining gap 14.5 cm. The initial depth already exceeds the block's height, so complete submersion is possible.