

Exercise walk-through

4.1

Sets, Relations and Functions

GAC Mathematics

1

The method

Method — Key knowledge: each operation depends on which elements are allowed

A **set** 集合 contains distinct elements; listing order and repeated entries do not change it. Union contains elements in either set, intersection elements in both. A **set complement** 补集 is relative to a specified universe U . A subset contains only elements of its parent set.

A relation pairs elements of two sets. A **function** 函数 assigns exactly one output to every input in its stated domain; several inputs may share an output. An inverse relation is a function on the original range only when each output comes from exactly one input. For finite sets, $|A \cup B| = |A| + |B| - |A \cap B|$.

Method — Worked example: subtract an overlap only once

Known: 30 learners, 18 study French, 15 German, and 7 both. The overlap is included in both subject totals.

$$|F \cup G| = |F| + |G| - |F \cap G| = 18 + 15 - 7 = 26$$

$$N_{\text{neither}} = |U| - |F \cup G| = 30 - 26 = 4$$

French-only is $18 - 7 = 11$ and German-only $15 - 7 = 8$. The four disjoint regions sum to 30.

Exercises 2.1-2.3

2.1 For $A = \{1, 2, 3\}$ and $B = \{3, 4\}$, list $A \cap B$ and $A \cup B$. [2]

2.2 Let $U = \{1, 2, 3, 4, 5, 6\}$ and $A = \{1, 3, 5\}$. List the complement of A in U and explain why 7 is not in it. [2]

2.3 A relation from domain $\{1, 2, 3\}$ to $\{a, b\}$ is $\{(1, a), (2, a), (3, b)\}$. State its range, decide whether it is a function, and justify the decision. [3]

Exercises 2.1-2.3

2.1 For $A = \{1, 2, 3\}$ and $B = \{3, 4\}$, list $A \cap B$ and $A \cup B$. [2]

2.1

Intersection is $\{3\}$. Union is $\{1, 2, 3, 4\}$. Do not repeat 3 in a set.

2.2 Let $U = \{1, 2, 3, 4, 5, 6\}$ and $A = \{1, 3, 5\}$. List the complement of A in U and explain why 7 is not in it. [2]

2.2

The complement is $U \setminus A = \{2, 4, 6\}$. Seven is outside the universe, so it is not in this complement.

2.3 A relation from domain $\{1, 2, 3\}$ to $\{a, b\}$ is $\{(1, a), (2, a), (3, b)\}$. State its range, decide whether it is a function, and justify the decision. [3]

2.3

Range is $\{a, b\}$. It is a function. Every domain input has exactly one output, although two inputs share a.

Exercise 2.4

2.4 Finite sets have $|A| = 15$, $|B| = 12$ and $|A \cap B| = 5$. Their universe has 30 elements. Find the union count and the number outside both sets. [3]

Exercise 2.4

2.4 Finite sets have $|A| = 15$, $|B| = 12$ and $|A \cap B| = 5$. Their universe has 30 elements. Find the union count and the number outside both sets. [3]

2.4

Use inclusion-exclusion.

$$|A \cup B| = |A| + |B| - |A \cap B| = 15 + 12 - 5 = 22$$

Union count 22.

$$N_{outside} = |U| - |A \cup B| = 30 - 22 = 8$$

Outside count 8.

Exercise 3.1

3.1 In a class of 40, 23 take music, 18 take drama and 7 take both. Find (a) music only, (b) drama only, (c) neither, and (d) exactly one of the two. Check your four-region totals. [6]

Exercise 3.1

3.1 In a class of 40, 23 take music, 18 take drama and 7 take both. Find (a) music only, (b) drama only, (c) neither, and (d) exactly one of the two. Check your four-region totals. [6]

3.1

Music only is $23 - 7 = 16$. Drama only is $18 - 7 = 11$. Inclusion-exclusion gives the union.

$$|M \cup D| = 23 + 18 - 7 = 34$$

Neither is $40 - 34 = 6$. Exactly one is $16 + 11 = 27$. The disjoint regions sum to $16 + 7 + 11 + 6 = 40$.

Exercise 3.2

3.2 Let $D = \{-2, -1, 0, 1, 2\}$ and define $f(x) = x^2$ on D . (a) List the range. (b) Explain

Exercise 3.2

3.2 Let $D = \{-2, -1, 0, 1, 2\}$ and define $f(x) = x^2$ on D . (a) List the range. (b) Explain

3.2

(a) Range is $\{0, 1, 4\}$.

(b) f is a function because each input has exactly one square. It is not one-to-one. Distinct inputs such as negative 2 and 2 both give 4.

(c) The inverse relation is not a function on the original range because 1 and 4 each correspond to two domain values.

(d) The restricted function is one-to-one and has the same range $\{0, 1, 4\}$. Its inverse is the nonnegative square root on that range. Checks: 0 maps back to 0, 1 to 1, and 4 to 2. The restriction is essential; do not silently discard the original negative inputs.