

Exercise walk-through

2.6

Statistics – Numerical Measures of Central Locations

GAC Mathematics

1

The method

Method — Key knowledge: the question determines which summary is useful

The **mean** 平均数 is sum divided by count. A frequency table uses $\bar{x} = \sum fx / \sum f$.

The **median** 中位数 is the middle ordered value, or the mean of the middle two for an even count. The **mode** 众数 is the most frequent value; there can be several modes or no unique mode.

A weighted mean uses weights proportional to the relevant contributions. Do not average group means equally unless their group sizes are equal or equal weighting is explicitly intended. An extreme value can move the mean more than the median, but no measure is automatically best for every question. Report units and describe what the chosen summary represents.

Method — Worked example: group size determines a combined mean

Known: group A has 10 scores with mean 60; group B has 20 with mean 75. Recover each group's total before combining.

$$\bar{x} = \frac{n_A \bar{x}_A + n_B \bar{x}_B}{n_A + n_B}$$

$$\bar{x} = \frac{10(60) + 20(75)}{10 + 20} = 70$$

The unweighted mean of 60 and 75 would be 67.5, which ignores the unequal counts.

Exercises 2.1-2.2

2.1 Find the mean of 4, 6, 8 and 10.

[2]

2.2 Find the median and mode of 2, 3, 3, 7, 9.

[2]

Exercises 2.1·2.2

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2.1

Add the values and divide by the count.

$$\bar{x} = \frac{\sum x}{n} = \frac{4 + 6 + 8 + 10}{4} = 7$$

Mean 7.

2.2 Find the median and mode of 2, 3, 3, 7, 9. [2]

2.2

The middle of the five ordered values is 3, so median 3. The most frequent value is also 3, so mode 3.

Exercise 2.3

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Multiply each value by its frequency.

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$\bar{x} = \frac{1(2) + 2(3) + 4(1)}{2 + 3 + 1}$$

Correct weighted total and count.

$$\bar{x} = \frac{12}{6} = 2$$

Mean 2.

Exercise 2.4

2.4 The ordered data are 2, 4, 6, 8. Find the median. Then replace 8 by 80 and find the new median and new mean. [3]

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Original median $m = (4 + 6)/2 = 5$. New median remains 5 because the middle two values are unchanged.

$$\bar{x}_{new} = \frac{2 + 4 + 6 + 80}{4} = 23$$

New mean 23.

Exercise 3.1

3.1 Group A has 8 learners with mean score 65. Group B has 12 learners with mean score 75. (a) Find the combined mean. (b) A single missing score is then added, bringing the combined mean for 21 learners to 70. Find that score. (c) Explain why averaging 65 and 75 equally is not the correct first calculation. [6]

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3.1

(a) Recover totals from each group.

$$T = n_A \bar{x}_A + n_B \bar{x}_B = 8(65) + 12(75) = 1420$$

Correct total.

Combined mean 71.

(b) Subtract the old total from the required new total.

$$x = 21(70) - 1420 = 50$$

Missing score 50. Equal weighting would give each group the same influence despite unequal learner counts.

Exercise 3.2

3.2 Five supplied monthly incomes are 1000, 1100, 1200, 1300 and 6400 units. (a) Find the mean and median. (b) A report says, “Most people earn the mean.” Explain what the summaries do and do not show. (c) If the highest income changes to 8400 while the others stay fixed, find the new mean and median. Which measure changes more, and why?

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(a) Mean uses all five incomes.

$$\bar{x} = \frac{1000 + 1100 + 1200 + 1300 + 6400}{5} = 2200$$

Mean 2200. Median 1200.

(b) Nobody in the supplied list earns 2200; the mean is a total-per-person summary, not a statement that most incomes equal it. The median marks the ordered middle and can

Full solution 3.2

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(b) Nobody in the supplied list earns 2200; the mean is a total-per-person summary, not a statement that most incomes equal it. The median marks the ordered middle and can describe the centre without claiming all or most incomes equal it.

(c) Recalculate the total after the change.

$$\bar{x}_{new} = \frac{1000 + 1100 + 1200 + 1300 + 8400}{5} = 2600$$

New mean 2600 and median still 1200. The mean rises by 400 because it includes the changed magnitude, while the ordered middle remains unchanged. These five invented records do not represent an entire population.