

Exercise walk-through

2.4

Probability – Key Concepts

GAC Mathematics

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The method

Method — Key knowledge: independence and mutual exclusion describe different relationships

A **sample space** 样本空间 lists possible outcomes. For equally likely outcomes, probability is favourable count divided by total count. The complement has probability $1 - P(A)$. For any events, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Events are **independent** 独立 when one does not change the probability of the other; then $P(A \cap B) = P(A)P(B)$. Mutually exclusive events cannot occur together.

Conditional probability 条件概率 uses $P(A | B) = P(A \cap B) / P(B)$ for $P(B) > 0$.

Sampling without replacement changes later probabilities. Ordered arrangements use permutations; unordered selections use combinations.

Method — Worked example: a second draw depends on the first

Known: two red and one blue counter, two random draws without replacement.
To get two reds, follow the red-red path and multiply its conditional probabilities.

$$P(RR) = P(R_1)P(R_2 \mid R_1)$$

$$P(RR) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

A first blue leaves only red counters; the blue-blue path has probability zero. The draws are not independent.

Exercises 2.1·2.2

2.1 A fair six-sided die is rolled once. Find the probability of a number greater than 4.[2]

2.2 If $P(A) = 0.35$, find the probability of the complement of A. [2]

Exercises 2.1·2.2

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2.1

Favourable outcomes are 5 and 6 among six equally likely outcomes.

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

Probability one third.

2.2 If $P(A) = 0.35$, find the probability of the complement of A. [2]

2.2

Complement probabilities sum to 1.

$$P(A^c) = 1 - P(A) = 1 - 0.35 = 0.65$$

Probability 0.65.

Exercise 2.3

2.3 Events have $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$. Find $P(A \cup B)$ and decide whether they are independent. [3]

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2.3

Subtract the double-counted overlap.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.2 = 0.7$$

Union probability 0.7. They are independent because $P(A)P(B) = 0.4(0.5) = 0.2 = P(A \cap B)$.

Exercise 2.4

2.4 Five distinct students stand in a line. (a) Count the possible orders. (b) Count the possible unordered teams of two drawn from them. [3]

3 Exam-style questions

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3 Exam-style questions

2.4

Order matters for a line.

$$N_{line} = 5! = 5(4)(3)(2)(1) = 120$$

120 orders. For an unordered pair, divide ordered choices by two:

$$N_{team} = \binom{5}{2} = \frac{5(4)}{2(1)} = 10$$

10 teams.

Exercise 3.1

3.1 A bag has 3 red and 2 blue counters. Two are drawn randomly without replacement. Find (a) the probability both are red and (b) the probability exactly one is red. Explain why the second draw cannot use the original proportions unchanged. [6]

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3.1

(a) The first red leaves two red among four counters.

$$P(RR) = P(R_1)P(R_2 \mid R_1) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$$

Both-red probability.

(b) Exactly one red has disjoint paths RB and BR.

Full solution 3.1 (1/2)

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(b) Exactly one red has disjoint paths RB and BR.

Full solution 3.1 (2/2)

$$P(RB) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$$

$$P(BR) = \frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$$

Correct path products.

$$P(\text{exactly one red}) = P(RB) + P(BR) = \frac{3}{10} + \frac{3}{10} = \frac{3}{5}$$

Final probability. The remaining counts depend on which colour was removed, so the draws are not independent.

Exercise 3.2

3.2 A class has 40 students. Of 24 who use the bus, 6 arrived late today. Of 16 who do not use the bus, 2 arrived late. Select one class member uniformly at random. Find (a) the probability of being late, (b) the probability of being late given bus use, and (c) the probability of bus use given being late. (d) Decide whether bus use and lateness are independent in this table. Explain why the table does not prove that bus use causes lateness. [8]

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(a) Add late counts and divide by the class size.

$$P(L) = \frac{6 + 2}{40} = 0.20$$

Late probability.

Full solution 3.2 (1/2)

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Late probability.

(b) Restrict the denominator to bus users.

Full solution 3.2 (2/2)

$$P(L \mid B) = \frac{6}{24} = 0.25$$

Conditional probability.

(c) Restrict the denominator to late students.

$$P(B \mid L) = \frac{6}{6 + 2} = 0.75$$

Conditional probability.

(d) Compare $P(L \mid B)$ with $P(L)$, or compare the joint probability with the product. They differ, so the events are not independent in the supplied table. This observational one-day table does not control other factors or randomly assign transport; association alone does not establish causation.