

Exercise walk-through

2.3

Finance – Investing Money and Loans

GAC Mathematics

1

The method

Method — Key knowledge: a rate has a period and a loan balance changes after payments

Principal 本金 is the initial amount invested or borrowed. **Simple interest** 单利 uses $I = Prt$ at a stated rate r per period and time t in matching periods. **Compound interest** 复利 uses $A = P(1 + r)^n$ when the rate and compounding period match and no other cash flows apply.

For the loan convention used here, interest is added to the opening balance and a payment is then subtracted: $B_n = B_{n-1}(1 + r) - M$. A final payment may need adjustment to avoid a negative balance. Compare total payments and fees under the same stated schedule. All examples use invented classroom terms, with no actual lending or investment advice.

Method — Worked example: interest then payment, in that order

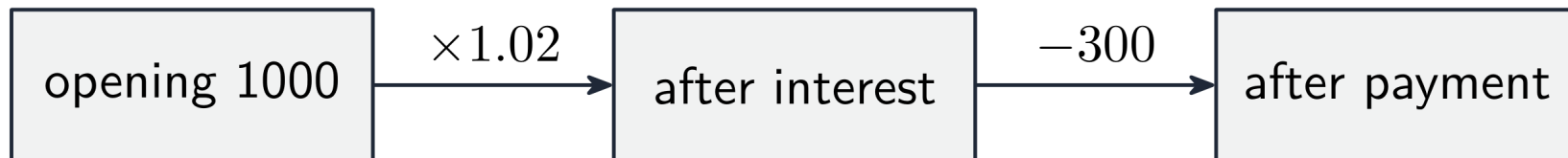
Known: opening balance 1000 units, period rate 2%, end-of-period payment 300 units.
Interest comes before payment under this stated convention.

$$B_1 = B_0(1 + r) - M$$

$$B_1 = 1000(1.02) - 300 = 720$$

The next period's interest uses 720, not the original principal.

$$B_2 = B_1(1 + r) - M = 720(1.02) - 300 = 434.40$$



Exercises 2.1·2.2

2.1 Find the simple interest on 800 units at 5% per year for 3 years.

[2]

2.2 Find the value of 800 units after 2 years at 5% compounded annually, with no other cash flows.

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Exercises 2.1·2.2

2.1 Find the simple interest on 800 units at 5% per year for 3 years.

[2]

2.1

Rate and time both use years.

$$I = Prt = 800(0.05)(3) = 120$$

Interest 120 units.

2.2 Find the value of 800 units after 2 years at 5% compounded annually, with no other cash flows.

[2]

2.2

Apply two annual multipliers.

$$A = P(1 + r)^n = 800(1.05)^2 = 882$$

Value 882 units.

Exercise 2.3

2.3 A nominal teaching rate is 12% per year, compounded monthly at 1% per month. Find the value of 1000 units after 6 months, to 2 decimal places. [3]

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2.3

Use the specified monthly rate with six monthly periods.

$$A = P(1 + r_m)^n$$

$$A = 1000(1.01)^6$$

Correct matching rate and period.

$$A = 1000(1.01)^6 \approx 1061.52$$

Value 1061.52 units.

Exercise 2.4

2.4 A loan opens at 600 units. Each month adds 1% interest before a 100-unit payment. Find the balances after the first two months. [3]

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2.4

Use the stated interest-before-payment recurrence.

$$B_n = B_{n-1}(1 + r) - M$$

$$B_1 = 600(1.01) - 100 = 506$$

First balance 506.

$$B_2 = 506(1.01) - 100 = 411.06$$

Second balance 411.06.

Exercise 3.1

3.1 Two investments start with 2000 units. A pays simple interest at 6% per year for 3 years. B pays 5.5% compounded annually for 3 years. There are no charges or other cash flows. Find each final value and the difference to 2 decimal places. State which final value is higher under these assumptions. [6]

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3.1

$$A_S = P(1 + rt) = 2000(1 + 0.06(3)) = 2360$$

A value 2360. B uses repeated multiplication.

$$A_C = P(1 + r)^n = 2000(1.055)^3 = 2348.48275$$

B value 2348.48 to two decimal places.

$$\Delta = A_S - A_C = 2360 - 2348.48275 \approx 11.52$$

Difference 11.52. A's final value is higher under the specified terms. This does not compare risk, availability or any unstated conditions.

Exercise 3.2

3.2 A 1000-unit loan adds 2% interest per period before payment. Payments of 300 are made at the end of the first three periods. At the end of period 4, pay exactly the amount still owed after that period interest. (a) Find the balances after periods 1, 2 and 3. (b) Find the adjusted final payment. (c) Find total paid and total interest. No fees apply.[8]

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3.2

Use the recurrence with interest first.

$$B_n = B_{n-1}(1 + r) - M$$

$$B_1 = 1000(1.02) - 300 = 720$$

First balance.

Full solution 3.2 (1/2)

3.2

Use the recurrence with interest first.

$$B_n = B_{n-1}(1 + r) - M$$

$$B_1 = 1000(1.02) - 300 = 720$$

First balance.

$$B_2 = 720(1.02) - 300 = 434.40$$

Second balance.

$$B_3 = 434.40(1.02) - 300 = 143.088$$

Full solution 3.2 (2/2)

Third balance. Period 4's final payment includes its interest.

$$M_4 = B_3(1 + r) = 143.088(1.02) = 145.94976 \approx 145.95$$

Final payment 145.95.

$$T = 3M + M_4 = 3(300) + 145.94976 \approx 1045.95$$

Total paid 1045.95.

$$I = T - P = 1045.94976 - 1000 \approx 45.95$$

Total interest 45.95. Carry unrounded balances through this mathematical model; an actual contract may specify a different rounding rule.