

Exercise walk-through

2.1

Sequence and Series

GAC Mathematics

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The method

Method — Key knowledge: constant differences and constant ratios give different models

An **arithmetic sequence** 等差数列 has constant difference d , with $u_n = a + (n - 1)d$. A **geometric sequence** 等比数列 has constant ratio r , with $u_n = ar^{n-1}$. Here a is the first term and n starts at 1. A **series** 级数 adds terms of a sequence. For n arithmetic terms, $S_n = n[2a + (n - 1)d]/2$. For geometric terms with $r \neq 1$, $S_n = a(1 - r^n)/(1 - r)$. If $r = 1$, the sum is na . For a nonzero first term, an infinite geometric sum is $S_\infty = a/(1 - r)$ only when $|r| < 1$; a finite sum can still exist when the infinite series does not converge.

Method — Worked example: row counts differ by a constant amount

Known: row counts are 4, 7 and 10, continuing with the same difference. Thus $a = 4$ and $d = 3$. For the fifth row:

$$u_n = a + (n - 1)d$$

$$u_5 = 4 + (5 - 1)(3) = 16$$

For the first five rows together, use the arithmetic sum, not the fifth term.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_5 = \frac{5}{2}[2(4) + (5 - 1)(3)] = 50$$

Exercise 2.1

2.1 An arithmetic sequence begins 5, 9, 13. Find its tenth term.

[2]

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The constant difference is 4. Use the n th-term formula.

$$u_n = a + (n - 1)d$$

$$u_{10} = 5 + (10 - 1)(4) = 41$$

Tenth term 41.

Exercise 2.2

2.2 A geometric sequence begins 3, 6, 12. Find its sixth term.

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2.2

The constant ratio is 2.

$$u_n = ar^{n-1}$$

$$u_6 = 3(2)^{6-1} = 96$$

Sixth term 96.

Exercise 2.3

2.3 Find the sum of the first 20 terms of the arithmetic sequence with first term 7 and common difference 3. [3]

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A sum includes every term, not only the last.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{20} = \frac{20}{2}[2(7) + (20 - 1)(3)]$$

Correct substitution.

$$S_{20} = 10(14 + 57) = 710$$

Sum 710.

Exercise 2.4

2.4 Find the sum of the first five terms of the geometric sequence with $a = 16$ and $r = 1/2$. State its infinite sum. [3]

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Use the finite geometric sum.

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_5 = \frac{16(1 - (1/2)^5)}{1 - 1/2} = 31$$

Exercise 3.1

3.1 An auditorium has 12 seats in its first row and 2 more seats in each subsequent row.

(a) Find the seat count in row 15. (b) Find the total in the first 15 rows. (c) Find the smallest number of complete rows whose combined count is at least 400. [6]

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(a) Row counts form an arithmetic sequence.

$$u_n = a + (n - 1)d$$

$$u_{15} = 12 + (15 - 1)(2) = 40$$

Row 15 contains 40 seats.

(b) Add row counts.

Full solution 3.1 (1/2)

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(b) Add row counts.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{15} = \frac{15}{2}[24 + 28] = 390$$

Full solution 3.1 (2/2)

Total 390.

(c) Express the total as a function of n .

$$S_n = \frac{n}{2}[24 + 2(n - 1)] = n(n + 11)$$

Check consecutive complete-row totals.

$$S_{15} = 15(26) = 390$$

$$S_{16} = 16(27) = 432$$

The totals increase for positive n , so the smallest count is 16 rows.

Exercise 3.2

3.2 A modelled sequence of daily deliveries is 80, 60, 45, continuing geometrically. Fractional values are allowed in this mathematical model. (a) Find its ratio and fifth term. (b) Find the sum of the first five terms. (c) Find the infinite sum and explain why it exceeds the five-term sum. (d) A different sequence is 10, 12, 14.4, continuing geometrically. Explain whether it has a finite infinite sum. [8]

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3.2

(a) Ratio is $r = u_2/u_1 = 60/80 = 0.75$.

$$u_n = ar^{n-1}$$

$$u_5 = 80(0.75)^4 = 25.3125$$

Fifth term.

Full solution 3.2 (1/2)

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Fifth term.

(b) Use the geometric sum.

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Full solution 3.2 (2/2)

$$S_5 = \frac{80(1 - 0.75^5)}{1 - 0.75} = 244.0625$$

Five-term sum.

(c) The ratio satisfies $|r| < 1$.

$$S_\infty = \frac{a}{1 - r} = \frac{80}{1 - 0.75} = 320$$

Infinite sum 320. The remaining terms are positive, so adding them increases the sum.

(d) The second ratio is 1.2, so its positive terms grow and $|r| < 1$ fails; there is no finite infinite sum. An infinite-horizon model is not a promise about actual deliveries.