

Exercise walk-through

1.5

Trigonometry

GAC Mathematics

1

The method

Method — Key knowledge: the known sides and angles determine which rule to use

In a right triangle, sine is opposite divided by hypotenuse; cosine is adjacent divided by hypotenuse; tangent is opposite divided by adjacent. Opposite and adjacent depend on the chosen angle. Set calculator angle mode to match the data: degrees and radians are different measures, with $180^\circ = \pi$ radians.

The **sine rule** 正弦定理 pairs each side with its opposite angle. Side-side-angle data can produce two possible triangles; do not assume an inverse sine gives every valid answer. The **cosine rule** 余弦定理 works with two sides and their included angle, or three sides. A **bearing** 方位角 is measured clockwise from north and written with three digits in degrees. Angle of elevation is measured upward from a horizontal line.

Method — Worked example: height from a horizontal distance and angle

Known: horizontal distance $d = 12$ m and elevation $\theta = 30^\circ$. Height h is opposite and d is adjacent, so use tangent in degree mode.

$$\tan \theta = \frac{h}{d}$$

$$h = d \tan \theta = 12 \tan 30^\circ \approx 6.93 \text{ m}$$

This is the height above the observer's horizontal sight level, not automatically the height above the ground.

Exercises 2.1-2.2

2.1 A right triangle has opposite side 6 cm and hypotenuse 10 cm for an acute angle θ . Find θ to 1 decimal place. [2]

2.2 Convert 150° to radians, giving an exact answer in terms of π . [2]

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2.1

Use sine because opposite and hypotenuse are given.

$$\theta = \sin^{-1} \left(\frac{o}{h} \right) = \sin^{-1} \left(\frac{6}{10} \right) \approx 36.9^\circ$$

Angle 36.9 degrees.

2.2 Convert 150° to radians, giving an exact answer in terms of pi. [2]

2.2

Multiply the degree measure by pi over 180.

$$\theta_{rad} = \theta_{deg} \frac{\pi}{180} = 150 \frac{\pi}{180} = \frac{5\pi}{6}$$

Exact answer $5\pi/6$ radians.

Exercise 2.3

2.3 In triangle ABC, $A = 40^\circ$, $B = 65^\circ$ and side a opposite A is 8 cm. Find side b opposite B to 3 significant figures. [3]

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2.3

Pair each side with its opposite angle.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$b = \frac{a \sin B}{\sin A} = \frac{8 \sin 65^\circ}{\sin 40^\circ}$$

Correct substitution or rearrangement.

$$b = \frac{8 \sin 65^\circ}{\sin 40^\circ} \approx 11.3 \text{ cm}$$

Correct length.

Exercise 2.4

2.4 Two sides of a triangle are 7 and 9 cm with included angle 60° . Find the third side to 3 significant figures. [3]

3 Exam-style questions

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2.4

Use cosine rule because the angle lies between the known sides.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c = \sqrt{a^2 + b^2 - 2ab \cos C} = \sqrt{7^2 + 9^2 - 2(7)(9) \cos 60^\circ}$$

Correct substitution.

$$c = \sqrt{7^2 + 9^2 - 2(7)(9) \cos 60^\circ} = \sqrt{67} \approx 8.19 \text{ cm}$$

Correct third side.

Exercise 3.1

3.1 A walker travels 4 km due east, then 3 km due north. (a) Find the straight-line distance from the start. (b) Find the bearing of the finishing point from the start, to the nearest degree. (c) Find the reverse bearing, also to the nearest degree. [6]

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3.1

(a) The east and north components are perpendicular.

$$d = \sqrt{e^2 + n^2} = \sqrt{4^2 + 3^2} = 5 \text{ km}$$

Distance 5 km.

(b) Measure clockwise from north, so east is opposite and north adjacent.

$$\beta = \tan^{-1} \left(\frac{e}{n} \right) = \tan^{-1} \left(\frac{4}{3} \right) \approx 53.13^\circ$$

Full solution 3.1

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Bearing 053 degrees.

(c) A reverse direction differs by 180 degrees.

$$\beta_{reverse} = \beta + 180^\circ \approx 53.13^\circ + 180^\circ = 233.13^\circ$$

Reverse bearing 233 degrees.

Exercise 3.2

3.2 An observer on level ground measures a tower elevation of 35° . After moving 10 m straight towards the tower, the elevation is 50° . The instrument remains 1.5 m above the ground. Let d be the initial horizontal distance to the tower base and h the tower height above the instrument. (a) Form two equations. (b) Find d and the total tower height to 3 significant figures. (c) Explain why the instrument height is added only once. [8]

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3.2

The near distance is $d - 10$ m. Both triangles share height h above instrument level.

$$h = d \tan 35^\circ$$

First equation.

$$h = (d - 10) \tan 50^\circ$$

Full solution 3.2 (1/2)

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$$h = d \tan 35^\circ$$

First equation.

$$h = (d - 10) \tan 50^\circ$$

Second equation. Equate the two expressions for h .

$$d \tan 35^\circ = (d - 10) \tan 50^\circ$$

Rearrange to isolate d .

$$, \quad 10 \tan 50^\circ$$

Full solution 3.2 (2/2)

$$d = \frac{10 \tan 50^\circ}{\tan 50^\circ - \tan 35^\circ}$$

$$d = \frac{10 \tan 50^\circ}{\tan 50^\circ - \tan 35^\circ} \approx 24.2 \text{ m}$$

Correct initial distance. Use the unrounded distance to find h .

$$h = d \tan 35^\circ \approx 16.9765 \text{ m}$$

$$H = h + 1.5 \approx 16.9765 + 1.5 = 18.5 \text{ m} \quad (3 \text{ s.f.})$$

Total height. Both observations use the same instrument level; 1.5 m is one offset from that shared horizontal line to the ground.