

Exercise walk-through

1.4

Geometry: Plane, Solid, and Euclidean Geometry

GAC Mathematics

1

The method

Method — Key knowledge: choose the correct dimension and distinguish surface from volume

Similar 相似 shapes have equal corresponding angles and proportional corresponding lengths. A length scale factor k produces area factor k^2 and volume factor k^3 for similar solids. Congruent shapes have the same size as well as shape.

In a right triangle, the **hypotenuse** 斜边 is opposite the right angle and $c^2 = a^2 + b^2$. A triangle's area uses perpendicular height, not a sloping side. **Volume** 体积 measures three-dimensional space in cubic units; surface area measures the boundary in square units. For a prism, volume is cross-sectional area times length. Keep all lengths in compatible units before calculating.

Method — Worked example: a triangular prism has a constant cross-section

Known: perpendicular triangle sides are 3 and 4 cm; prism length is 10 cm. Find the cross-sectional area, then multiply by the prism length.

$$A = \frac{1}{2}bh = \frac{1}{2}(3)(4) = 6 \text{ cm}^2$$

$$V = AL = 6(10) = 60 \text{ cm}^3$$

The sloping triangle side is not its perpendicular height. Volume is not the area of the triangular end.

Exercises 2.1·2.2

2.1 A right triangle has perpendicular sides 5 and 12 cm. Find its hypotenuse. [2]

2.2 A triangle has base 8 cm and perpendicular height 6 cm. Find its area. [2]

Exercises 2.1·2.2

2.1 A right triangle has perpendicular sides 5 and 12 cm. Find its hypotenuse. [2]

2.1

Use Pythagoras because the given sides meet at a right angle.

$$c = \sqrt{a^2 + b^2} = \sqrt{5^2 + 12^2} = 13 \text{ cm}$$

Hypotenuse 13 cm.

2.2 A triangle has base 8 cm and perpendicular height 6 cm. Find its area. [2]

2.2

Use half the rectangle area.

$$A = \frac{1}{2}bh = \frac{1}{2}(8)(6) = 24 \text{ cm}^2$$

Area 24 square centimetres.

Exercise 2.3

2.3 Two similar triangles have corresponding lengths 4 and 10 cm. The smaller has area 12 cm^2 . Find the length scale factor and the larger area. [3]

Exercise 2.3

2.3 Two similar triangles have corresponding lengths 4 and 10 cm. The smaller has area 12 cm^2 . Find the length scale factor and the larger area. [3]

2.3

The larger-to-smaller length factor is $k = 10/4 = 2.5$. Areas scale by the square of the length factor.

$$A_L = k^2 A_S = (2.5)^2 (12) = 75 \text{ cm}^2$$

Larger area 75 square centimetres.

Exercise 2.4

2.4 A closed cylinder has radius 3 cm and height 8 cm. Find its volume, giving an exact answer in terms of π and a decimal to 3 significant figures. [3]

Exercise 2.4

2.4 A closed cylinder has radius 3 cm and height 8 cm. Find its volume, giving an exact answer in terms of pi and a decimal to 3 significant figures. [3]

2.4

Use circular area times height.

$$V = \pi r^2 h = \pi(3)^2(8) = 72\pi \text{ cm}^3$$

Exact answer.

$$V = 72\pi \text{ cm}^3 \approx 226 \text{ cm}^3$$

Correct 3 significant figures.

Exercise 3.1

3.1 A rectangular block measures 8 by 5 by 3 cm. (a) Find its volume and total surface area. (b) A geometrically similar block has all lengths doubled. Find its volume and explain why the volume is not merely doubled. [6]

Exercise 3.1

3.1 A rectangular block measures 8 by 5 by 3 cm. (a) Find its volume and total surface area. (b) A geometrically similar block has all lengths doubled. Find its volume and explain why the volume is not merely doubled. [6]

3.1

Volume uses all three perpendicular lengths.

$$V = lwh = 8(5)(3) = 120 \text{ cm}^3$$

Volume. Sum the three pairs of opposite faces.

$$S = 2(lw + lh + wh)$$

$$S = 2(8(5) + 8(3) + 5(3)) = 158 \text{ cm}^2$$

Surface area.

$$V_L = k^3 V_S = 2^3(120) = 960 \text{ cm}^3$$

Larger volume. Each of the three dimensions doubles, so the product grows eightfold.

Exercise 3.2

3.2 An open rectangular tank has internal length 80 cm, width 50 cm and depth 40 cm.

(a) Find its capacity in litres, using $1000 \text{ cm}^3 = 1 \text{ litre}$. (b) It initially contains water to depth 25 cm. Find the water volume. (c) A rectangular solid block measuring 20 by 10 by 10 cm is fully submerged without spilling; it does not absorb water. Find the new water depth and state the remaining depth below the rim. [8]

Exercise 3.2

3.2 An open rectangular tank has internal length 80 cm, width 50 cm and depth 40 cm.

(a) Find its capacity in litres, using $1000 \text{ cm}^3 = 1 \text{ litre}$. (b) It initially contains water to depth 25 cm. Find the water volume. (c) A rectangular solid block measuring 20 by 10 by 10 cm is fully submerged without spilling; it does not absorb water. Find the new water depth and state the remaining depth below the rim. [8]

3.2

Capacity is internal rectangular volume.

$$V_T = lwh = 80(50)(40) = 160000 \text{ cm}^3$$

$$C = \frac{V_T}{1000} = \frac{160000}{1000} = 160 \text{ litres}$$

Full solution 3.2 (1/2)

3.2

Capacity is internal rectangular volume.

$$V_T = lwh = 80(50)(40) = 160000 \text{ cm}^3$$

$$C = \frac{V_T}{1000} = \frac{160000}{1000} = 160 \text{ litres}$$

Capacity 160 litres.

$$V_W = lwh_W = 80(50)(25) = 100000 \text{ cm}^3 = 100 \text{ litres}$$

Initial water volume 100 litres. The submerged block displaces its own volume.

$$V_B = l_B w_B h_B = 20(10)(10) = 2000 \text{ cm}^3$$

Full solution 3.2 (2/2)

Block volume.

$$\Delta h = \frac{V_B}{lw} = \frac{2000}{80(50)} = 0.5 \text{ cm}$$

Correct rise.

$$h_{new} = h_W + \Delta h = 25 + 0.5 = 25.5 \text{ cm}$$

New depth.

$$h_{gap} = h_T - h_{new} = 40 - 25.5 = 14.5 \text{ cm}$$

Remaining gap 14.5 cm. The initial depth already exceeds the block's height, so complete submersion is possible.