

Exercise walk-through

1.3

Algebra: Graphs of Algebraic Relations and Coordinate Geometry

GAC Mathematics

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The method

Method — Key knowledge: coordinates make geometric relationships testable

For a nonvertical straight line, **gradient** 斜率 is change in y divided by change in x . In $y = mx + c$, m is the gradient and c is the y -intercept. A vertical line $x = k$ has undefined gradient. Nonvertical parallel lines have equal gradients. Two nonvertical perpendicular lines have gradients whose product is negative 1; horizontal and vertical lines are also perpendicular.

The **midpoint** 中点 averages each coordinate. Distance uses Pythagoras with the coordinate differences. A quadratic graph has a **vertex** 顶点 at its turning point. Its real roots are x -coordinates where it meets the x -axis; a repeated root touches rather than crosses. Solving two graph equations finds their intersections.

Method — Worked example: a rising line through two given points

Known: $A = (1, 2)$ and $B = (4, 8)$. Use gradient because the line is nonvertical.

$$m = \frac{y_B - y_A}{x_B - x_A} = \frac{8 - 2}{4 - 1} = 2$$

The line equation is $y = mx + c$. Substitute A to find the intercept.

$$c = y_A - mx_A = 2 - 2(1) = 0$$

Thus $y = 2x$. The diagram's horizontal change is 3 and vertical change is 6.

Exercises 2.1·2.2

2.1 Find the gradient of the line through $(2, 1)$ and $(6, 9)$.

[2]

2.2 Find the midpoint of $(-2, 5)$ and $(4, -1)$.

[2]

Exercises 2.1·2.2

2.1 Find the gradient of the line through $(2, 1)$ and $(6, 9)$.

[2]

2.1

Use coordinate differences in the same order.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 1}{6 - 2} = 2$$

Gradient 2.

2.2 Find the midpoint of $(-2, 5)$ and $(4, -1)$.

[2]

2.2

Average the corresponding coordinates.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-2 + 4}{2}, \frac{5 - 1}{2} \right) = (1, 2)$$

Midpoint $(1, 2)$.

Exercise 2.3

2.3 Find the equation of the line with gradient 3 through $(2, 7)$. Give its y-intercept. [3]

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2.3

Use $y = mx + c$ with the known point.

$$c = y - mx = 7 - 3(2) = 1$$

$$y = 3x + 1$$

Correct equation. Y-intercept 1, or point (0, 1).

Exercise 2.4

2.4 For $y = (x - 2)^2 - 9$, sketch the graph, labelling the vertex and both x-intercepts.

3 Exam-style questions

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Vertex form gives vertex $(2, -9)$. At an x-intercept y is zero.

$$(x - 2)^2 = 9 \quad \implies \quad x - 2 = \pm 3$$

$$x = 2 \pm 3 = -1 \text{ or } 5$$

Intercepts $(-1, 0)$ and $(5, 0)$. An upward-opening parabola through these intercepts, with the labelled vertex below the axis, earns the final point.

Exercise 3.1

3.1 Let $A = (1, 2)$ and $B = (7, 10)$. Find (a) the distance AB , (b) its midpoint, and (c) the equation of the perpendicular bisector of AB . [6]

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3.1

(a) Distance uses coordinate differences.

$$d = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(7 - 1)^2 + (10 - 2)^2} = 10$$

Distance 10.

(b) Average coordinates.

$$M = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right) = \left(\frac{1 + 7}{2}, \frac{2 + 10}{2} \right) = (4, 6)$$

Full solution 3.1 (1/2)

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Full solution 3.1 (2/2)

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Midpoint $(4, 6)$.

(c) The original gradient is $m_{AB} = (10 - 2)/(7 - 1) = 4/3$. Use its negative reciprocal.

$$m_{\text{perp}} = -\frac{1}{m_{AB}} = -\frac{3}{4}$$

The bisector passes through the midpoint.

$$c = y_M - m_{\text{perp}}x_M = 6 - (-3/4)(4) = 9$$

$$y = -\frac{3}{4}x + 9$$

Correct equation.

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At an intersection both y-values agree.

$$x^2 - 3x + 1 = x + 1 \implies x^2 - 4x = 0$$

$$x(x - 4) = 0$$

Correct factorisation. Thus $x = 0$ or $x = 4$.

$$y = x + 1 \implies y_1 = 0 + 1 = 1, \quad y_2 = 4 + 1 = 5$$

Intersections $(0, 1)$ and $(4, 5)$.

Full solution on next slide

Full solution 3.2 (1/2)

3.2

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Full solution 3.2 (2/2)

$$y = x + 1 \implies y_1 = 0 + 1 = 1, \quad y_2 = 4 + 1 = 5$$

Intersections $(0, 1)$ and $(4, 5)$.

Complete the square or use $x_v = -b/(2a)$.

$$y = (x - 3/2)^2 - 5/4$$

Vertex $(3/2, -5/4)$.

Below the line means $x(x - 4) < 0$, which holds between the roots: $0 < x < 4$. The endpoints give equal y-values, so strict inequality excludes them.