

Exercise walk-through

1.2

Algebra I: Introductory Algebra

GAC Mathematics

1

The method

Method — Key knowledge: an equation stays balanced when both sides undergo the same valid operation

An **expression** 表达式 has no equals sign; an **equation** 方程 states that two expressions are equal. Expand by multiplying every term in a bracket. Factorising reverses expansion. Collect only like terms.

Apply the same operation to both sides of an equation. Dividing by an unknown expression can lose a solution if it might be zero. For an **inequality** 不等式, multiplying or dividing both sides by a negative number reverses the sign. A quadratic can have two, one or no real roots. In simultaneous equations, a solution must satisfy both equations.

Method — Worked example: reverse the inequality sign when dividing by a negative

Known: solve $7 - 3x > 1$. Subtract 7 from both sides, then divide by negative 3. The last operation reverses the inequality.

$$7 - 3x > 1 \implies -3x > -6$$

$$x < \frac{-6}{-3} = 2$$

The open circle excludes 2. Check $x = 1$: $7 - 3(1) = 4 > 1$; the boundary $x = 2$ gives equality and is excluded.

Exercise 2.1

2.1 Expand and simplify $3(2x - 5) + 4x$.

[2]

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Distribute 3 to both terms.

$$E = 3(2x - 5) + 4x = 6x - 15 + 4x$$

$$E = 10x - 15$$

Correct simplified expression.

Exercise 2.2

2.2 Solve $5x - 7 = 18$.

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Add 7 to both sides, then divide by 5.

$$5x = 18 + 7 = 25$$

$$x = \frac{25}{5} = 5$$

Correct solution.

Exercise 2.3

2.3 Factorise $x^2 - x - 12$, then solve $x^2 - x - 12 = 0$.

[3]

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2.3

Two numbers with product negative 12 and sum negative 1 are negative 4 and 3.

$$x^2 - x - 12 = (x - 4)(x + 3)$$

Correct factorisation. A zero product means at least one factor is zero.

$$x - 4 = 0 \text{ or } x + 3 = 0$$

$$x = 4 \text{ or } x = -3$$

Both roots.

Exercise 2.4

2.4 Solve $4 - 2x \leq 10$. State whether the boundary value is included.

[3]

3 Exam-style questions

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3 Exam-style questions

2.4

Subtract 4, then divide by negative 2 and reverse the sign.

$$-2x \leq 10 - 4 = 6$$

$$x \geq \frac{6}{-2} = -3$$

Correct inequality. Negative 3 is included because the original sign allows equality.

Exercise 3.1

3.1 A club buys 3 adult tickets and 2 student tickets for 46 units. Another group buys 2 adult and 4 student tickets for 44 units. Both groups use the same prices. Find each ticket price and check both totals. [6]

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3.1

Let a and s be the adult and student prices. Translate both totals.

$$3a + 2s = 46, \quad 2a + 4s = 44$$

Double the first equation so the student coefficients match.

$$6a + 4s = 92$$

Subtract the second equation.

Full solution 3.1 (1/2)

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Let a and s be the adult and student prices. Translate both totals.

$$3a + 2s = 46, \quad 2a + 4s = 44$$

Double the first equation so the student coefficients match.

$$6a + 4s = 92$$

Full solution 3.1 (2/2)

Subtract the second equation.

$$4a = 92 - 44 = 48$$

$$a = \frac{48}{4} = 12$$

Adult price 12.

$$s = \frac{46 - 3a}{2} = \frac{46 - 3(12)}{2} = 5$$

Student price 5. Checks: $3(12) + 2(5) = 46$ and $2(12) + 4(5) = 44$. Both checks earn B1.

Exercise 3.2

3.2 A rectangle has width x metres and length $(x + 3)$ metres. Its area is 40 m^2 . (a) Form and solve an equation for x . (b) Choose the physical solution and find the perimeter. (c) A second rectangle has the same width x but length $(x + 5)$ metres. Find the increase in area. [8]

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3.2

Area equals length times width.

$$A = lw$$

$$40 = x(x+3) \implies x^2 + 3x - 40 = 0$$

Correct equation. Factorise.

Full solution 3.2 (1/2)

3.2

Area equals length times width.

$$A = lw$$

$$40 = x(x + 3) \implies x^2 + 3x - 40 = 0$$

Correct equation. Factorise.

$$x^2 + 3x - 40 = (x + 8)(x - 5)$$

Full solution 3.2 (2/2)

$$x = -8 \text{ or } x = 5$$

Both roots. Choose $x = 5$ because a physical width must be positive.

$$P = 2(l + w) = 2((5 + 3) + 5) = 26 \text{ m}$$

Perimeter 26 m.

Valid comparison.

$$\Delta A = 5(5 + 5) - 5(5 + 3) = 50 - 40 = 10 \text{ m}^2$$

Increase 10 square metres.