

Exercise walk-through

1.1

Terminology and Arithmetic Review

GAC Mathematics

1

The method

Method — Key knowledge: operations and percentage changes need the right order and reference value

An **integer** 整数 is a whole number, including negative whole numbers and zero. A **rational number** 有理数 can be written as a ratio of integers with a nonzero denominator. Fractions need equivalent forms before addition; multiplying and dividing have equal priority and are worked left to right, as are addition and subtraction. A **percentage** 百分比 compares a part or change with a stated base. Increasing by $r\%$ uses multiplier $1 + r/100$; decreasing uses $1 - r/100$. Reversing a percentage change means dividing by its multiplier, not applying the same percentage to the new amount. Round the final result as instructed rather than losing precision early.

Method — Worked example: a decrease and increase use different bases

Known: an invented price of 80 units falls by 20%, then rises by 20%. Why use successive multipliers? Each change is calculated from the current price.

$$P_{new} = P_{old}(1 - r/100)$$

$$P_1 = 80(1 - 20/100) = 64$$

$$P_{new} = P_{old}(1 + r/100)$$

$$P_2 = 64(1 + 20/100) = 76.8$$

The price is not back at 80. The decrease was 16, while the later increase was 12.8, because the bases differ.

Exercise 2.1

2.1 Evaluate $18 - 3(4 - 7)$, showing the bracket step.

[2]

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2.1

Known: a bracket is multiplied before the subtraction.

$$V = 18 - 3(4 - 7) = 18 - 3(-3)$$

$$V = 18 - (-9) = 27$$

Final result 27.

Exercise 2.2

2.2 Calculate $\frac{3}{4} - \frac{2}{5}$ in simplest form.

[2]

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2.2

Use a common denominator of 20.

$$F = \frac{3}{4} - \frac{2}{5} = \frac{15}{20} - \frac{8}{20}$$

$$F = \frac{15 - 8}{20} = \frac{7}{20}$$

Final fraction 7/20.

Exercise 2.3

2.3 Find the highest common factor and lowest common multiple of 18 and 24. [3]

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Prime factors give the common and combined powers.

$$18 = 2 \times 3^2, \quad 24 = 2^3 \times 3$$

$$\text{HCF} = 2 \times 3 = 6$$

HCF 6.

$$\text{LCM} = 2^3 \times 3^2 = 72$$

LCM 72.

Exercise 2.4

2.4 A price rises from 120 to 150 units. Calculate the percentage increase and the percentage decrease needed to return from 150 to 120. [3]

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The change is 30, but the two bases differ.

$$r_{up} = \frac{P_{new} - P_{old}}{P_{old}} \times 100\%$$

$$r_{up} = \frac{150 - 120}{120} \times 100\% = 25\%$$

$$r_{down} = \frac{P_{old} - P_{new}}{P_{old}} \times 100\%$$

$$r_{down} = \frac{150 - 120}{150} \times 100\% = 20\%$$

Required decrease 20%.

Exercise 3.1

3.1 Two school bells ring every 12 and 18 minutes. They ring together at 09:00. (a) Find when they next ring together. (b) A supplier offers 48 pens for 36 units or 60 pens for 42 units. Compare unit prices and state which has the lower cost per pen. [6]

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3.1

(a) Simultaneous ringing repeats after the LCM.

$$12 = 2^2 \times 3, \quad 18 = 2 \times 3^2$$

$$t = \text{LCM}(12, 18) = 2^2 \times 3^2 = 36 \text{ min}$$

36 minutes. Next time 09:36.

(b) Divide cost by pen count.

Full solution 3.1

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(b) Divide cost by pen count.

$$u = \frac{C}{n}$$

$$u_A = \frac{36}{48} = 0.75, \quad u_B = \frac{42}{60} = 0.70$$

Both unit prices correct. The 60-pen offer is lower per pen; its total outlay is nevertheless higher.

Exercise 3.2

3.2 A club sells an item for 72 units after a 10% discount on its marked price. (a) Find the marked price. (b) A different offer takes 15% off the same marked price and then adds a 4-unit delivery charge. Find its total. (c) Compare the offers if the first has no delivery charge. Explain why adding 10% to 72 does not recover the marked price. [8]

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3.2

(a) Discount multiplier is 0.90.

$$P_{sale} = 0.90P_{marked}$$

$$P_{marked} = \frac{P_{sale}}{0.90}$$

Full solution 3.2 (1/2)

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(a) Discount multiplier is 0.90.

$$P_{sale} = 0.90P_{marked}$$

$$P_{marked} = \frac{P_{sale}}{0.90}$$

$$P_{marked} = \frac{72}{0.90} = 80$$

Marked price 80.

(b) Apply the 15% multiplier before adding delivery.

$$C = 0.85P_{marked} + D$$

Full solution 3.2 (2/2)

$$C = 0.85(80) + 4 = 68 + 4 = 72$$

Discounted item price 68. Delivered total 72.

(c) Totals are equal under the stated delivery assumptions. Adding 10% to the sale price uses 72 rather than the original base 80.

$$P' = 72(1 + 10/100) = 79.2 \neq 80$$

The numerical check or a clear inverse-multiplier explanation earns the final point.