Exercise walk-through

4.1
Sets, Relations and Functions

GAC Mathematics

Sets, relations and functions (2)

- A **relation** pairs elements of two sets. A **function** is a **relation** in which each input has exactly one output. "Each student $\to$ their birthday" is a **function** (two students may share a birthday); "each birthday $\to$ the students born on it" is not, because one input can have several outputs.
- A **Venn diagram** , introduced by the English logician John Venn in 1880, turns a set problem into a picture. Check that the regions add up to the total.

$$N_{neither} = |U| - |F \cup G| = 30 - 26 = 4$$

Counting systems

- A **number base** b uses the digits 0 to $b-1$, and each place is worth a power of b. **Decimal** uses ten, **binary** two, **hexadecimal** sixteen (digits 0–9 then A–F).
- Every digit's value is its **place value**: in binary the places are 1, 2, 4, 8, 16 and so on.
- **Hexadecimal** is shorthand for binary: one hex digit is exactly four bits. Web colours use it: ‘#FF8000‘ means red 255, green 128, blue 0, a bright orange.

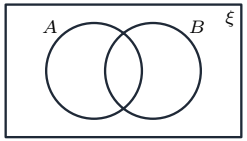
$$V = \sum d_i 2^i$$

Graphs and networks

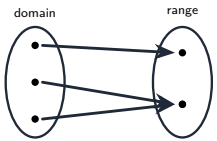
- A **graph** is a set of **vertices** joined by **edges**. It models anything with connections: roads, friendships, dependencies.
- The **degree** of a vertex is the number of **edges** that meet it. Every edge has two ends, so the degrees always add up to twice the number of edges. This is the fact behind Euler's Königsberg proof.
- A **tree** is a connected **graph** with no cycles; a **tree** with n vertices has $n - 1$ edges. A family tree and a computer's folder structure are trees.

Exercises 2.1·2.2

2.1 For $A = \{1, 2, 3\}$ and $B = \{3, 4\}$, list $A \cap B$ and $A \cup B$. [2]

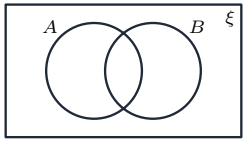


2.2 Let $U = \{1, 2, 3, 4, 5, 6\}$ and $A = \{1, 3, 5\}$. List the complement of A in U and explain why 7 is not in it. [2]



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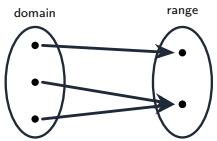
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2.1

Intersection is $\{3\}$. Union is $\{1, 2, 3, 4\}$. Do not repeat 3 in a set.

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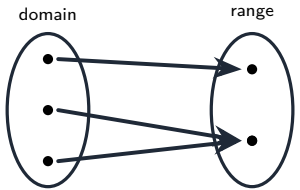


2.2

The complement is $U \setminus A = \{2, 4, 6\}$. Seven is outside the universe, so it is not in this complement.

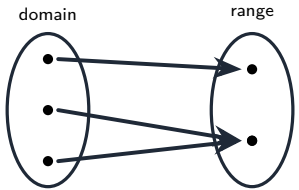
Exercise 2.3

2.3 A relation from domain $\{1, 2, 3\}$ to $\{a, b\}$ is $\{(1, a), (2, a), (3, b)\}$. State its range, decide whether it is a function, and justify the decision. [3]



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2.3

Range is $\{a, b\}$. It is a function. Every domain input has exactly one output, although two inputs share a.

Exercise 2.4

2.4 Finite sets have $|A| = 15$, $|B| = 12$ and $|A \cap B| = 5$. Their universe has 30 elements. Find the union count and the number outside both sets. [3]



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2.4

Use inclusion-exclusion.

$$|A \cup B| = |A| + |B| - |A \cap B| = 15 + 12 - 5 = 22$$

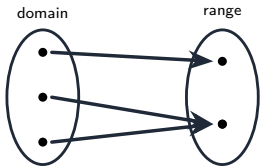
Union count 22.

$$N_{outside} = |U| - |A \cup B| = 30 - 22 = 8$$

Outside count 8.

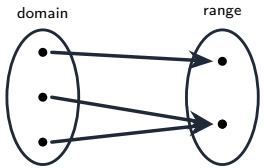
Exercise 3.1

3.1 In a class of 40, 23 take music, 18 take drama and 7 take both. Find (a) music only, (b) drama only, (c) neither, and (d) exactly one of the two. Check your four-region totals. [6]



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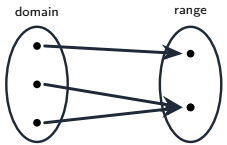
Music only is $23 - 7 = 16$. Drama only is $18 - 7 = 11$. Inclusion-exclusion gives the union.

$$|M \cup D| = 23 + 18 - 7 = 34$$

Neither is $40 - 34 = 6$. Exactly one is $16 + 11 = 27$. The disjoint regions sum to $16 + 7 + 11 + 6 = 40$.

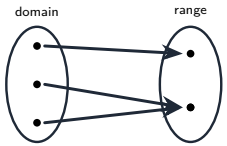
Exercise 3.2

3.2 Let $D = \{-2, -1, 0, 1, 2\}$ and define $f(x) = x^2$ on D. (a) List the range. (b) Explain whether f is a function and whether it is one-to-one. (c) Decide whether its inverse relation is a function from its range to D. (d) Restrict the domain to $D_+ = \{0, 1, 2\}$; describe an inverse function on the resulting range, and check every allowed output. [8]



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3.2

(a) Range is $\{0, 1, 4\}$.

(b) f is a function because each input has exactly one square. It is not one-to-one. Distinct inputs such as negative 2 and 2 both give 4.

(c) The inverse relation is not a function on the original range because 1 and 4 each correspond to two domain values.

(d) The restricted function is one-to-one and has the same range $\{0, 1, 4\}$. Its inverse is the nonnegative square root on that range. Checks: 0 maps back to 0, 1 to 1, and 4 to 2. The restriction is essential; do not silently discard the original negative inputs.