Exercise walk-through

3.1 Differentiation

GAC Mathematics

Differentiation

- The **power rule** : if $y = x^n$ then $\dfrac{dy}{dx} = nx^{n-1}$.
- The **product rule** : $(uv)' = u'v + uv'$.
- The **quotient rule** , where $v \neq 0$: $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Differentiation (2)

- The **chain rule** : if $y = f(g(x))$ then $\dfrac{dy}{dx} = f'(g(x)) \cdot g'(x)$.
- **Stationary points** occur where $f'(x) = 0$. The **second derivative** then classifies it: negative gives a local **maximum** , positive a local **minimum** . If the **second derivative** is zero, check the sign of $f'(x)$ on each side; $y = x^3$ at $x = 0$ is a stationary point that is neither.

$$m_{sec} = \frac{f(b) - f(a)}{b - a} = \frac{4 - 1}{2 - 1} = 3$$

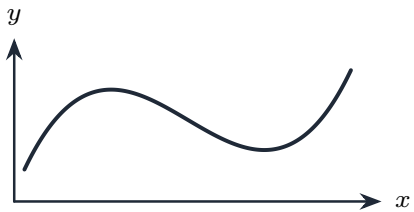
Integration

- An **indefinite integral** has no limits and needs the **constant of integration** : $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$ for $n \neq -1$.
- A **definite integral** has limits and gives a number: $\int_a^b f(x) \, dx = F(b) - F(a)$.
- **Integration by substitution** reverses the **chain rule**.
- △ Forgetting $+c$ on an **indefinite integral** loses marks on answers that are otherwise correct.

$$I = \int_{-1}^{1} x \, dx = \left[\frac{x^2}{2}\right]_{-1}^{1} = \frac{1}{2} - \frac{1}{2} = 0$$

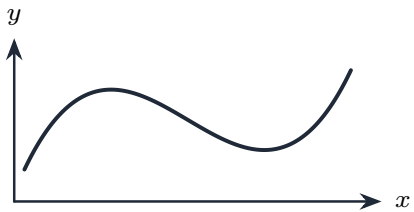
Exercise 2.1

2.1 Differentiate $y = 4x^3 - 6x^2 + 5$. [2]



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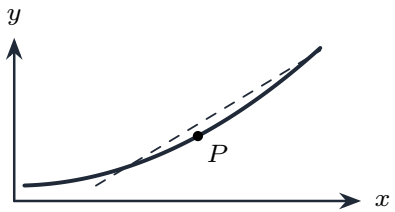
Apply the power rule term by term; a constant has derivative zero.

$$\frac{dy}{dx} = 12x^2 - 12x$$

Correct derivative.

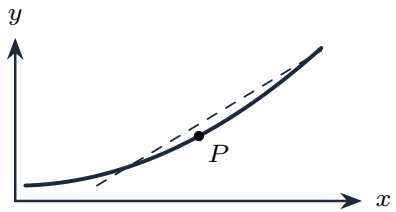
Exercise 2.2

2.2 Differentiate $y = (2x + 1)^3$. [2]



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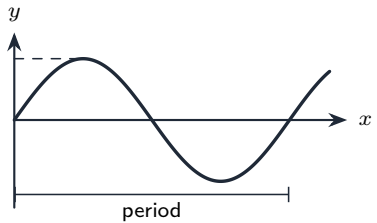
Use the chain rule, including inner derivative 2.

$$\frac{dy}{dx} = 3(2x + 1)^2(2) = 6(2x + 1)^2$$

Correct derivative.

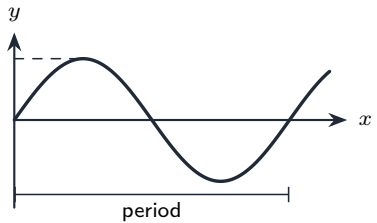
Exercise 2.3

2.3 Differentiate $y = x^2 \sin x$, where x is in radians. [3]



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2.3

Use the product rule.

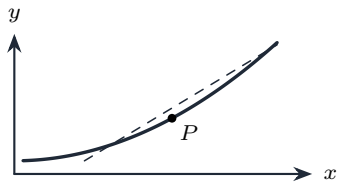
$$y' = u'v + uv'$$

$$y' = 2x \sin x + x^2 \cos x$$

Correct first product contribution. Correct second contribution and their sum.

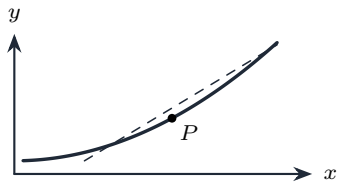
Exercise 2.4

2.4 For $y = (2x + 1)/(x - 1)$, state the excluded x-value and find the derivative. [3]



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2.4

Exclude $x = 1$ because the denominator is zero. Use the quotient rule.

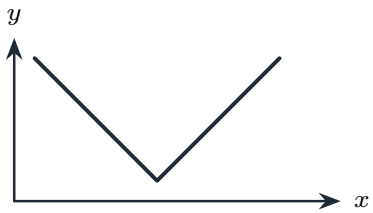
$$y' = \frac{u'v - uv'}{v^2}$$

$$y' = \frac{2(x-1) - (2x+1)}{(x-1)^2} = \frac{-3}{(x-1)^2}$$

Correct derivative on the stated domain.

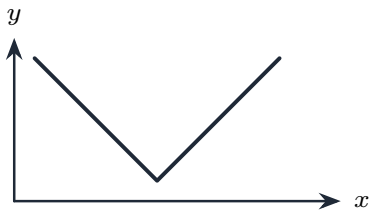
Exercise 3.1

3.1 Use difference quotients to examine $f(x) = |x|$ at x equal to 0. (a) Find the right-hand and left-hand limiting slopes. (b) Decide whether a derivative exists at 0. (c) Give the derivative for x greater than 0 and x less than 0. [6]



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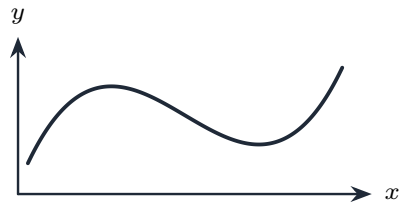


3.1

At zero, the difference quotient is $|h|/h$. For $h > 0$ it equals 1, so the right-hand limit is 1. For $h < 0$ it equals negative 1, so the left-hand limit is negative 1. The two one-sided limits differ, so no derivative exists at zero. For positive x, $f'(x) = 1$. For negative x, $f'(x) = -1$. The function itself is still continuous at zero.

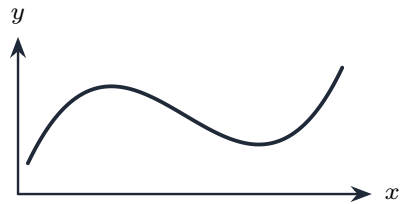
Exercise 3.2

3.2 For $f(x) = x^3 - 6x^2 + 9x + 1$, find and classify both stationary points. State all intervals on which the function is strictly increasing or strictly decreasing. [8]



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Differentiate.

$$f'(x) = 3x^2 - 12x + 9 = 3(x - 1)(x - 3)$$

Correct factorisation. Stationary x-values are 1 and 3.

Full solution on next slide

Full solution 3.2

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Differentiate.

$$f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$$

Correct factorisation. Stationary x-values are 1 and 3.

$$f(1) = 1 - 6 + 9 + 1 = 5$$

$$f(3) = 27 - 54 + 27 + 1 = 1$$

Points $(1, 5)$ and $(3, 1)$. Use the second derivative to classify them.

$$f''(x) = 6x - 12$$

At 1 it is negative 6, giving a local maximum; at 3 it is 6, giving a local minimum. Both classifications correct. The first derivative is positive for $x < 1$ and $x > 3$, so increasing on those intervals. It is negative for $1 < x < 3$, so decreasing there.