Exercise walk-through

2.4
Probability – Key Concepts

GAC Mathematics

Probability: key concepts (2)

- The **complement** rule, $P(\text{not } A) = 1 - P(A)$, is often the fastest route to an answer containing "at least one".
- **Counting techniques** give the number of outcomes: a **permutation** counts ordered selections, a **combination** counts unordered ones.
- For events that overlap, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; conditional probability is $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

$$P(RR) = P(R_1) \times P(R_2 \mid R_1) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

Statistics: measures of variability

- The **range** is the largest value minus the smallest, so one extreme value can change it a lot.
- The **interquartile range (IQR)** is $Q_3 - Q_1$, the spread of the middle half. A **box plot** shows the minimum, quartiles, **median** and maximum.
- The **standard deviation** measures spread using squared distances from the **mean**. For a whole **population**, $\sigma = \sqrt{\sum (x-\mu)^2/n}$; for a sample estimate, $s = \sqrt{\sum (x-\bar{x})^2/(n-1)}$. It has the same units as the data.
- Two data sets can share a mean and be completely different: a city with every day at 20 °C and one swinging between 0 °C and 40 °C have the same mean temperature.

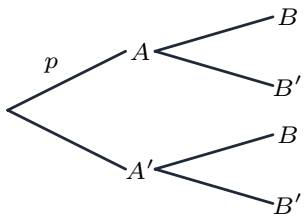
$$\text{range} = 12 - 2 = 10 \qquad IQR = 8 - 4 = 4$$

Sources for this lesson

- GAC010 Mathematics II course outline (sequences, finance, probability and statistics).
- *Labour Law of the People's Republic of China* (1994), Article 44, on overtime pay.
- National Bureau of Statistics of China, *Statistical Communiqué on the 2023 National Economic and Social Development* (per capita disposable income, **mean** and **median**).

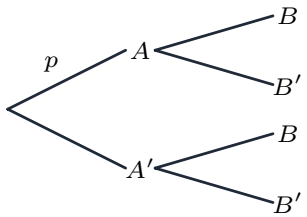
Exercise 2.1

2.1 A fair six-sided die is rolled once. Find the probability of a number greater than 4. [2]



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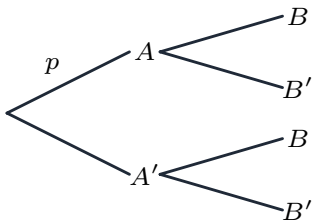
Favourable outcomes are 5 and 6 among six equally likely outcomes.

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

Probability one third.

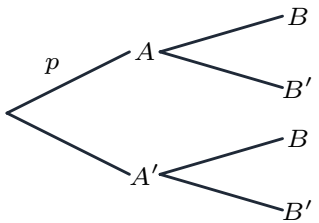
Exercise 2.2

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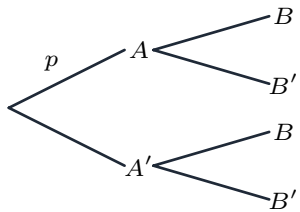
Complement probabilities sum to 1.

$$P(A^c) = 1 - P(A) = 1 - 0.35 = 0.65$$

Probability 0.65.

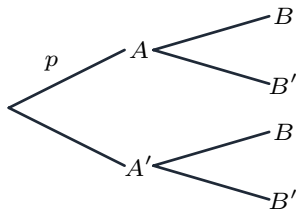
Exercise 2.3

2.3 Events have $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$. Find $P(A \cup B)$ and decide whether they are independent. [3]



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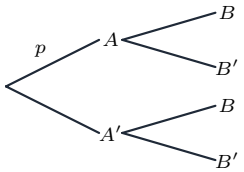
Subtract the double-counted overlap.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.2 = 0.7$$

Union probability 0.7. They are independent because $P(A)P(B) = 0.4(0.5) = 0.2 = P(A \cap B)$.

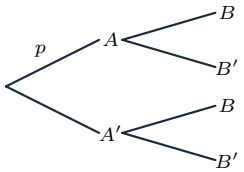
Exercise 2.4

2.4 Five distinct students stand in a line. (a) Count the possible orders. (b) Count the possible unordered teams of two drawn from them. [3]



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2.4

Order matters for a line.

$$N_{line} = 5! = 5(4)(3)(2)(1) = 120$$

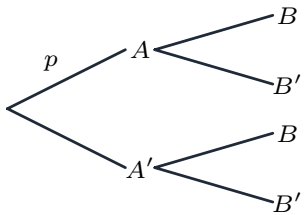
120 orders. For an unordered pair, divide ordered choices by two:

$$N_{team} = \binom{5}{2} = \frac{5(4)}{2(1)} = 10$$

10 teams.

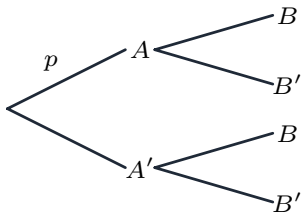
Exercise 3.1

3.1 A bag has 3 red and 2 blue counters. Two are drawn randomly without replacement. Find (a) the probability both are red and (b) the probability exactly one is red. Explain why the second draw cannot use the original proportions unchanged. [6]



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3.1

(a) The first red leaves two red among four counters.

$$P(RR) = P(R_1)P(R_2 \mid R_1) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$$

Both-red probability.

Full solution on next slide

Full solution 3.1 (1/2)

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Both-red probability.

Full solution 3.1 (2/2)

(b) Exactly one red has disjoint paths RB and BR.

$$P(RB) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$$

$$P(BR) = \frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$$

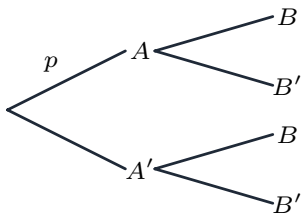
Correct path products.

$$P(\text{exactly one red}) = P(RB) + P(BR) = \frac{3}{10} + \frac{3}{10} = \frac{3}{5}$$

Final probability. The remaining counts depend on which colour was removed, so the draws are not independent.

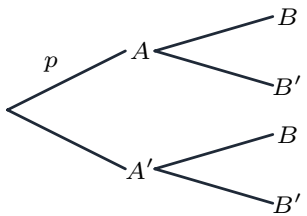
Exercise 3.2

3.2 A class has 40 students. Of 24 who use the bus, 6 arrived late today. Of 16 who do not use the bus, 2 arrived late. Select one class member uniformly at random. Find (a) the probability of being late, (b) the probability of being late given bus use, and (c) the probability of bus use given being late. (d) Decide whether bus use and lateness are independent in this table. Explain why the table does not prove that bus use causes lateness. [8]



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3.2

(a) Add late counts and divide by the class size.

Full solution on next slide

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Full solution 3.2 (1/2)

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(a) Add late counts and divide by the class size.

$$P(L) = \frac{6 + 2}{40} = 0.20$$

Late probability.

(b) Restrict the denominator to bus users.

$$P(L \mid B) = \frac{6}{24} = 0.25$$

Full solution 3.2 (2/2)

Conditional probability.

(c) Restrict the denominator to late students.

$$P(B \mid L) = \frac{6}{6+2} = 0.75$$

Conditional probability.

(d) Compare $P(L \mid B)$ with $P(L)$, or compare the joint probability with the product. They differ, so the events are not independent in the supplied table. This observational one-day table does not control other factors or randomly assign transport; association alone does not establish causation.