Exercise walk-through

2.3 Finance – Investing Money and Loans

GAC Mathematics

Statistics: collecting and displaying data

- A **population** is everyone you want to describe; a **sample** is who you actually measure.
- To generalise, the **sample** must be **representative**: chosen so that every part of the **population** has a fair chance of being included.
- **Qualitative data** are categories; **quantitative data** measure amounts, either **discrete** (counted) or **continuous** (measured).
- **Bias** enters through who you ask, who answers and how the question is worded. The *Literary Digest* poll had all three problems: its lists favoured the wealthy, and the 24% who replied were a self-selected group.

$$D_1 = \frac{20}{10} = 2 \qquad D_2 = \frac{20}{20} = 1$$

Statistics: measures of variability

- The **range** is the largest value minus the smallest, so one extreme value can change it a lot.
- The **interquartile range (IQR)** is $Q_3 - Q_1$, the spread of the middle half. A **box plot** shows the minimum, quartiles, **median** and maximum.
- The **standard deviation** measures spread using squared distances from the **mean**. For a whole **population**, $\sigma = \sqrt{\sum (x-\mu)^2/n}$; for a sample estimate, $s = \sqrt{\sum (x-\bar{x})^2/(n-1)}$. It has the same units as the data.
- Two data sets can share a mean and be completely different: a city with every day at 20 °C and one swinging between 0 °C and 40 °C have the same mean temperature.

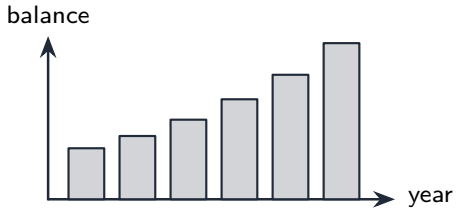
$$\text{range} = 12 - 2 = 10 \qquad IQR = 8 - 4 = 4$$

Sources for this lesson

- GAC010 Mathematics II course outline (sequences, finance, probability and statistics).
- *Labour Law of the People's Republic of China* (1994), Article 44, on overtime pay.
- National Bureau of Statistics of China, *Statistical Communiqué on the 2023 National Economic and Social Development* (per capita disposable income, **mean** and **median**).

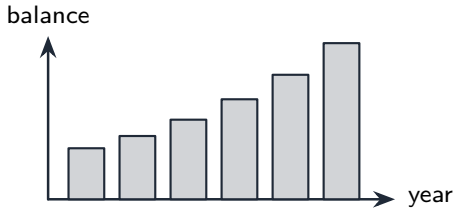
Exercise 2.1

2.1 Find the simple interest on 800 units at 5% per year for 3 years. [2]



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2.1

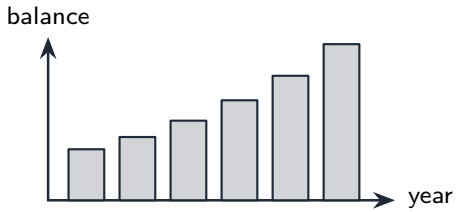
Rate and time both use years.

$$I = Prt = 800(0.05)(3) = 120$$

Interest 120 units.

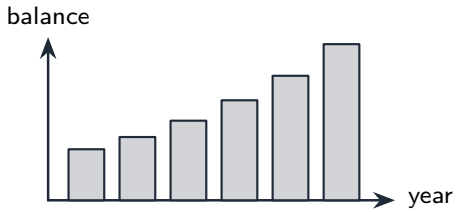
Exercise 2.2

2.2 Find the value of 800 units after 2 years at 5% compounded annually, with no other cash flows. [2]



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2.2

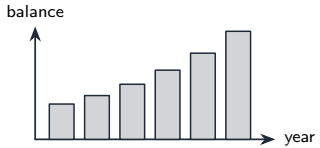
Apply two annual multipliers.

$$A = P(1 + r)^n = 800(1.05)^2 = 882$$

Value 882 units.

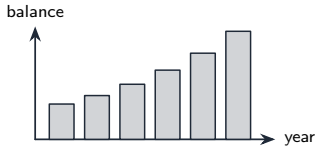
Exercise 2.3

2.3 A nominal teaching rate is 12% per year, compounded monthly at 1% per month. Find the value of 1000 units after 6 months, to 2 decimal places. [3]



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2.3

Use the specified monthly rate with six monthly periods.

$$A = P(1 + r_m)^n$$

$$A = 1000(1.01)^6$$

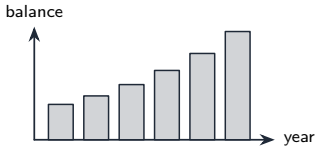
Correct matching rate and period.

$$A = 1000(1.01)^6 \approx 1061.52$$

Value 1061.52 units.

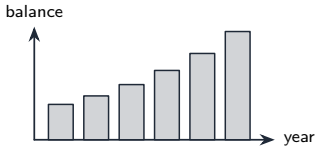
Exercise 2.4

2.4 A loan opens at 600 units. Each month adds 1% interest before a 100-unit payment. Find the balances after the first two months. [3]



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2.4

Use the stated interest-before-payment recurrence.

$$B_n = B_{n-1}(1 + r) - M$$

$$B_1 = 600(1.01) - 100 = 506$$

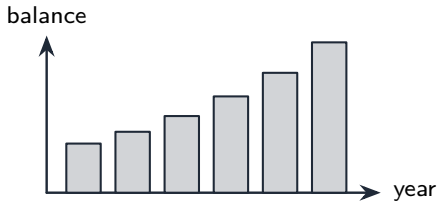
First balance 506.

$$B_2 = 506(1.01) - 100 = 411.06$$

Second balance 411.06.

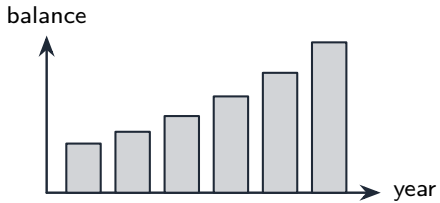
Exercise 3.1

3.1 Two investments start with 2000 units. A pays simple interest at 6% per year for 3 years. B pays 5.5% compounded annually for 3 years. There are no charges or other cash flows. Find each final value and the difference to 2 decimal places. State which final value is higher under these assumptions. [6]



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3.1

A adds simple interest to principal.

$$A_S = P(1 + rt) = 2000(1 + 0.06(3)) = 2360$$

Full solution on next slide

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Full solution 3.1

3.1

A adds simple interest to principal.

$$A_S = P(1 + rt) = 2000(1 + 0.06(3)) = 2360$$

A value 2360. B uses repeated multiplication.

$$A_C = P(1 + r)^n = 2000(1.055)^3 = 2348.48275$$

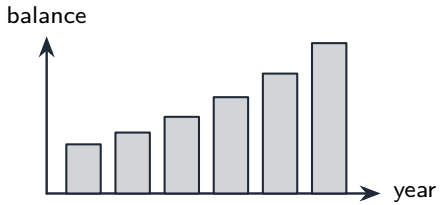
B value 2348.48 to two decimal places.

$$\Delta = A_S - A_C = 2360 - 2348.48275 \approx 11.52$$

Difference 11.52. A's final value is higher under the specified terms. This does not compare risk, availability or any unstated conditions.

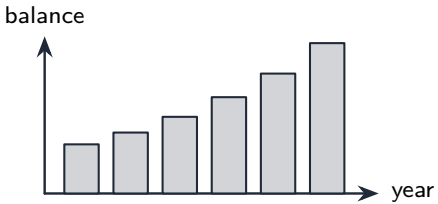
Exercise 3.2

3.2 A 1000-unit loan adds 2% interest per period before payment. Payments of 300 are made at the end of the first three periods. At the end of period 4, pay exactly the amount still owed after that period interest. (a) Find the balances after periods 1, 2 and 3. (b) Find the adjusted final payment. (c) Find total paid and total interest. No fees apply. [8]



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3.2

Use the recurrence with interest first.

$$B_n = B_{n-1}(1 + r) - M$$

Full solution on next slide

Full solution 3.2 (1/2)

3.2

Use the recurrence with interest first.

$$B_n = B_{n-1}(1 + r) - M$$

$$B_1 = 1000(1.02) - 300 = 720$$

First balance.

$$B_2 = 720(1.02) - 300 = 434.40$$

Second balance.

$$B_3 = 434.40(1.02) - 300 = 143.088$$

Third balance. Period 4's final payment includes its interest.

Full solution 3.2 (2/2)

$$M_4 = B_3(1 + r) = 143.088(1.02) = 145.94976 \approx 145.95$$

Final payment 145.95.

$$T = 3M + M_4 = 3(300) + 145.94976 \approx 1045.95$$

Total paid 1045.95.

$$I = T - P = 1045.94976 - 1000 \approx 45.95$$

Total interest 45.95. Carry unrounded balances through this mathematical model; an actual contract may specify a different rounding rule.