Exercise walk-through

2.2
Finance – Earning Money and Purchasing Goods

GAC Mathematics

Statistics: collecting and displaying data

- A **population** is everyone you want to describe; a **sample** is who you actually measure.
- To generalise, the **sample** must be **representative**: chosen so that every part of the **population** has a fair chance of being included.
- **Qualitative data** are categories; **quantitative data** measure amounts, either **discrete** (counted) or **continuous** (measured).
- **Bias** enters through who you ask, who answers and how the question is worded. The *Literary Digest* poll had all three problems: its lists favoured the wealthy, and the 24% who replied were a self-selected group.

$$D_1 = \frac{20}{10} = 2 \qquad D_2 = \frac{20}{20} = 1$$

Statistics: measures of variability

- The **range** is the largest value minus the smallest, so one extreme value can change it a lot.
- The **interquartile range (IQR)** is $Q_3 - Q_1$, the spread of the middle half. A **box plot** shows the minimum, quartiles, **median** and maximum.
- The **standard deviation** measures spread using squared distances from the **mean**. For a whole **population**, $\sigma = \sqrt{\sum (x-\mu)^2/n}$; for a sample estimate, $s = \sqrt{\sum (x-\bar{x})^2/(n-1)}$. It has the same units as the data.
- Two data sets can share a mean and be completely different: a city with every day at 20 °C and one swinging between 0 °C and 40 °C have the same mean temperature.

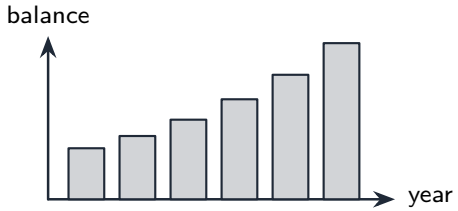
$$\text{range} = 12 - 2 = 10 \qquad IQR = 8 - 4 = 4$$

Sources for this lesson

- GAC010 Mathematics II course outline (sequences, finance, probability and statistics).
- *Labour Law of the People's Republic of China* (1994), Article 44, on overtime pay.
- National Bureau of Statistics of China, *Statistical Communiqué on the 2023 National Economic and Social Development* (per capita disposable income, **mean** and **median**).

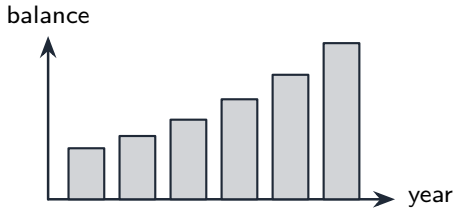
Exercise 2.1

2.1 A worker earns 18 units per hour for 7 hours. Find gross pay. [2]



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2.1

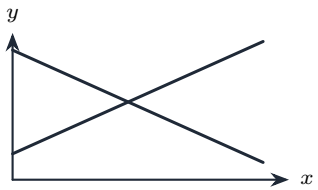
Multiply the rate by hours worked.

$$G = ph = 18(7) = 126 \text{ units}$$

Gross pay 126.

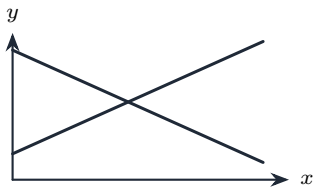
Exercise 2.2

2.2 A pack of 12 notebooks costs 30 units. A pack of 20 costs 46 units. Compare cost per notebook. [2]



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2.2

Divide each total by its count.

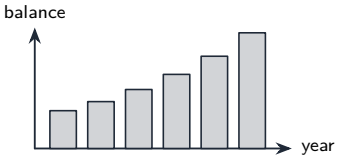
$$u_A = C_A / n_A = 30/12 = 2.50$$

$$u_B = C_B / n_B = 46/20 = 2.30$$

The 20-pack costs less per notebook. Both prices and the conclusion earn A1.

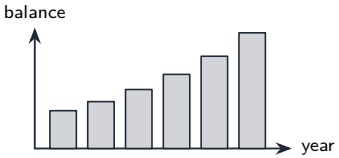
Exercise 2.3

2.3 A 200-unit item receives a 15% discount. A delivery fee of 6 units is then added. Find the delivered total and the overall saving compared with 200 units, as a percentage of 200. [3]



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2.3

Apply the discount to the item before adding the fee.

$$T = P(1 - r) + F = 200(0.85) + 6 = 176$$

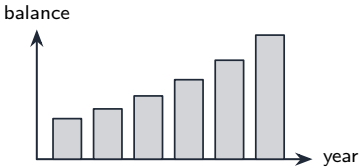
Total 176.

$$s = \frac{P - T}{P} \times 100\% = \frac{200 - 176}{200} \times 100\% = 12\%$$

Overall saving 12%.

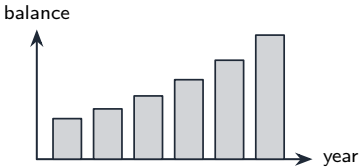
Exercise 2.4

2.4 An invented pay rule deducts 10% of gross pay plus a fixed 8 units. If gross pay is 360 units, find the total deduction and net pay. [3]



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2.4

Calculate the percentage and fixed parts.

$$D = rG + F = 0.10(360) + 8 = 44$$

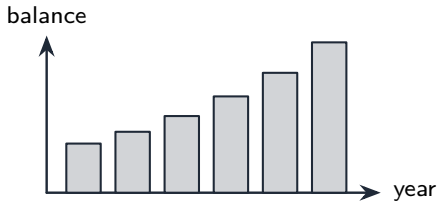
Deduction 44.

$$N = G - D = 360 - 44 = 316$$

Net pay 316.

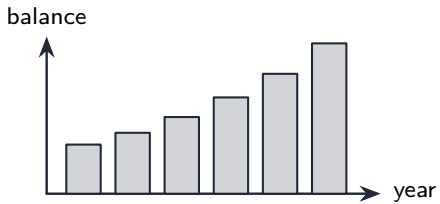
Exercise 3.1

3.1 A worker completes 35 regular hours at 16 units per hour and 5 overtime hours at 1.5 times the regular rate. Commission is 3% of all sales of 1200 units. A fixed deduction of 22 units applies once. Find regular pay, overtime pay, commission, gross pay and net pay. [6]



Exercise 3.1

3.1 A worker completes 35 regular hours at 16 units per hour and 5 overtime hours at 1.5 times the regular rate. Commission is 3% of all sales of 1200 units. A fixed deduction of 22 units applies once. Find regular pay, overtime pay, commission, gross pay and net pay. [6]



3.1

Regular pay uses regular hours.

$$R = h_r p = 35(16) = 560$$

Full solution on next slide

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Full solution 3.1 (1/2)

3.1

Regular pay uses regular hours.

$$R = h_r p = 35(16) = 560$$

Regular pay 560.

$$O = h_o k p = 5(1.5)(16) = 120$$

Overtime pay 120.

Full solution 3.1 (2/2)

$$C = rS = 0.03(1200) = 36$$

Commission 36.

$$G = R + O + C = 560 + 120 + 36 = 716$$

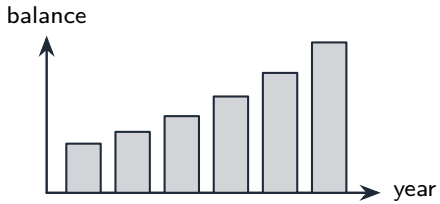
Gross pay 716.

$$N = G - D = 716 - 22 = 694$$

Net pay 694.

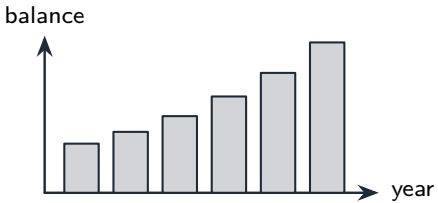
Exercise 3.2

3.2 Shop A sells a 500-unit item with a 20% discount and then a 30-unit delivery fee. Shop B sells the identical item with successive 10% discounts and free delivery. (a) Find each total and the difference. (b) Explain why two 10% discounts are not 20% off. (c) Find the largest delivery fee Shop A could charge while costing no more than Shop B. Assume no other charges. [8]



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3.2

(a) Discount A before adding delivery.

$$T_A = P(1 - r_A) + F = 500(0.80) + 30 = 430$$

Full solution on next slide

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Full solution 3.2 (1/2)

3.2

(a) Discount A before adding delivery.

$$T_A = P(1 - r_A) + F = 500(0.80) + 30 = 430$$

A total 430. Apply B's multipliers successively.

$$T_B = P(1 - r_1)(1 - r_2) = 500(0.90)(0.90) = 405$$

B total 405.

Full solution 3.2 (2/2)

$$\Delta = T_A - T_B = 430 - 405 = 25$$

B is cheaper by 25.

(b) The second discount uses the reduced base; the combined multiplier is 0.81, a 19% reduction.

(c) A must satisfy $400 + F \le 405$.

$$F \le T_B - 0.80P = 405 - 400 = 5$$

Largest delivery fee 5 units. The conclusion depends on the stated identical item and absence of other charges.