Exercise walk-through

2.1 Sequence and Series

GAC Mathematics

Finance: earning money and purchasing goods

- **Gross pay** is what you earn; **net pay** is what arrives after **deductions** such as tax and social insurance.
- **Simple interest** is $I = Prt$: interest on the original amount only.
- **Compound interest** is $A = P(1 + r)^n$: interest is added to the balance and then earns interest itself, so the balances form a **geometric sequence**.

$$G = 8(20) + 2(1.5)(20) = 160 + 60 = ¥220$$

Finance: investing money and loans

- The **present value** of a future amount, at a constant rate r per period, is $PV = \dfrac{FV}{(1+r)^n}$: how much you would need to invest now to have FV later.
- An **annuity** is a **series** of equal payments at regular intervals.
- In a **loan repayment** schedule, each period interest is added to the balance and the payment is taken off. The order matters.
- An **APR** is an annual rate. If interest is added monthly, the rate per month is the **APR** divided by 12, and the effective yearly growth is higher: a nominal 12% with 1% a month gives $(1.01)^{12} - 1 \approx 12.68\%$.

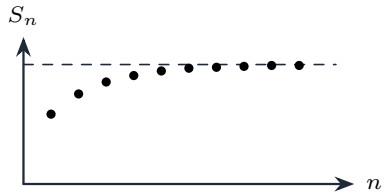
$$B_1 = 1000(1.02) - 300 = ¥720$$

Sources for this lesson

- GAC010 Mathematics II course outline (sequences, finance, probability and statistics).
- *Labour Law of the People's Republic of China* (1994), Article 44, on overtime pay.
- National Bureau of Statistics of China, *Statistical Communiqué on the 2023 National Economic and Social Development* (per capita disposable income, **mean** and **median**).

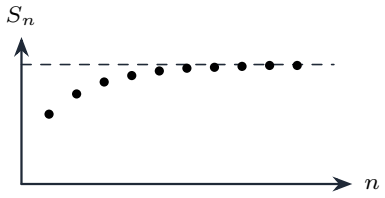
Exercise 2.1

2.1 An arithmetic sequence begins 5, 9, 13. Find its tenth term. [2]



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2.1

The constant difference is 4. Use the nth-term formula.

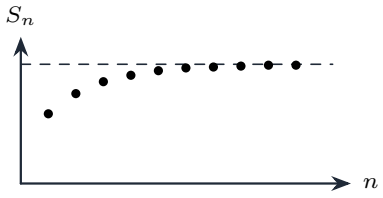
$$u_n = a + (n-1)d$$

$$u_{10} = 5 + (10-1)(4) = 41$$

Tenth term 41.

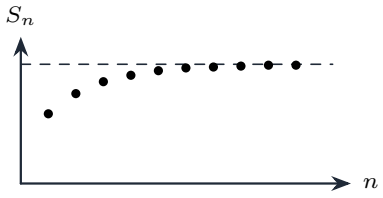
Exercise 2.2

2.2 A geometric sequence begins 3, 6, 12. Find its sixth term. [2]



Exercise 2.2

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2.2

The constant ratio is 2.

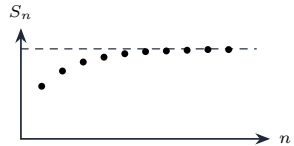
$$u_n = ar^{n-1}$$

$$u_6 = 3(2)^{6-1} = 96$$

Sixth term 96.

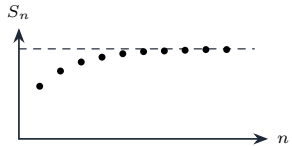
Exercise 2.3

2.3 Find the sum of the first 20 terms of the arithmetic sequence with first term 7 and common difference 3. [3]



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2.3

A sum includes every term, not only the last.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2(7) + (20-1)(3)]$$

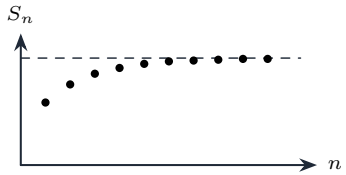
Correct substitution.

$$S_{20} = 10(14 + 57) = 710$$

Sum 710.

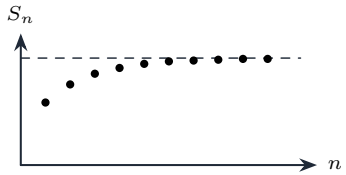
Exercise 2.4

2.4 Find the sum of the first five terms of the geometric sequence with $a = 16$ and $r = 1/2$. State its infinite sum. [3]



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2.4

Use the finite geometric sum.

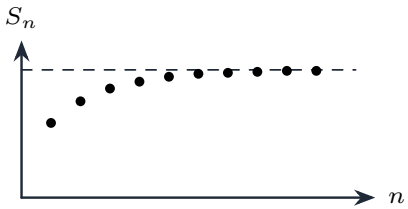
$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_5 = \frac{16(1 - (1/2)^5)}{1 - 1/2} = 31$$

Finite sum 31. Since $|r| < 1$, $S_\infty = a/(1 - r) = 16/(1/2) = 32$.

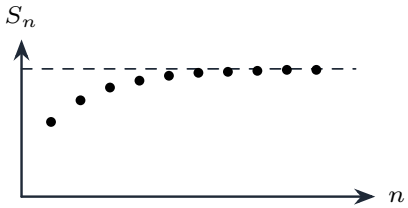
Exercise 3.1

3.1 An auditorium has 12 seats in its first row and 2 more seats in each subsequent row. (a) Find the seat count in row 15. (b) Find the total in the first 15 rows. (c) Find the smallest number of complete rows whose combined count is at least 400. [6]



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3.1

(a) Row counts form an arithmetic sequence.

$$u_n = a + (n-1)d$$

Full solution on next slide

Full solution 3.1 (1/2)

3.1

(a) Row counts form an arithmetic sequence.

$$u_n = a + (n-1)d$$

$$u_{15} = 12 + (15-1)(2) = 40$$

Row 15 contains 40 seats.

(b) Add row counts.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[24 + 28] = 390$$

Total 390.

Full solution 3.1 (2/2)

(c) Express the total as a function of n.

$$S_n = \frac{n}{2}[24 + 2(n-1)] = n(n+11)$$

Check consecutive complete-row totals.

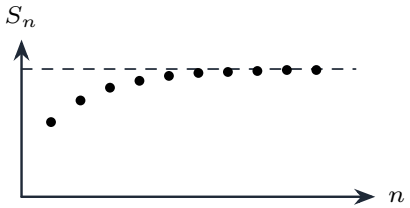
$$S_{15} = 15(26) = 390$$

$$S_{16} = 16(27) = 432$$

The totals increase for positive n, so the smallest count is 16 rows.

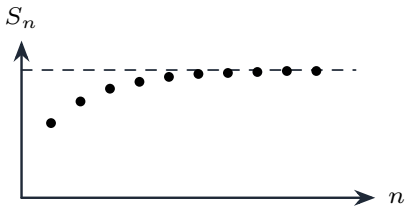
Exercise 3.2

3.2 A modelled sequence of daily deliveries is 80, 60, 45, continuing geometrically. Fractional values are allowed in this mathematical model. (a) Find its ratio and fifth term. (b) Find the sum of the first five terms. (c) Find the infinite sum and explain why it exceeds the five-term sum. (d) A different sequence is 10, 12, 14.4, continuing geometrically. Explain whether it has a finite infinite sum. [8]



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3.2

(a) Ratio is $r = u_2/u_1 = 60/80 = 0.75$.

$$u_n = ar^{n-1}$$

Full solution on next slide

Full solution 3.2 (1/2)

3.2

(a) Ratio is $r = u_2/u_1 = 60/80 = 0.75$.

$$u_n = ar^{n-1}$$

$$u_5 = 80(0.75)^4 = 25.3125$$

Fifth term.

(b) Use the geometric sum.

Full solution 3.2 (2/2)

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_5 = \frac{80(1 - 0.75^5)}{1 - 0.75} = 244.0625$$

Five-term sum.

(c) The ratio satisfies $|r| < 1$.

$$S_\infty = \frac{a}{1 - r} = \frac{80}{1 - 0.75} = 320$$

Infinite sum 320. The remaining terms are positive, so adding them increases the sum.

(d) The second ratio is 1.2, so its positive terms grow and $|r| < 1$ fails; there is no finite infinite sum. An infinite-horizon model is not a promise about actual deliveries.