Exercise walk-through

1.5 Trigonometry

GAC Mathematics

Algebra I: introductory algebra

- To solve a **linear equation** , do the same operation to both sides until the letter stands alone.
- A **quadratic equation** has the form $ax^2 + bx + c = 0$ with $a \neq 0$. Factorise when you can, or use $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- **Simultaneous equations** are two equations in two unknowns, solved by substitution or elimination. Two straight lines meet at one point, or never (parallel), or everywhere (the same line).
- △ An inequality **reverses** when you multiply or divide both sides by a negative number: from $-2x > 6$ it follows that $x < -3$.

$$(x+3)(x-5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15$$

Algebra: graphs and coordinate geometry

- Lines are **parallel** when their gradients are equal, and **perpendicular** when their gradients multiply to -1.
- The **midpoint** of a segment is the average of the endpoints; the **distance** between two points comes from Pythagoras: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- A quadratic graph is a **parabola**. Its **vertex** is the turning point; its **roots** are where it meets the x-axis. The path of a thrown ball is close to a parabola.
- To solve two equations **graphically**, draw both and read where they cross; check the answer algebraically, because a drawing is only approximate.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

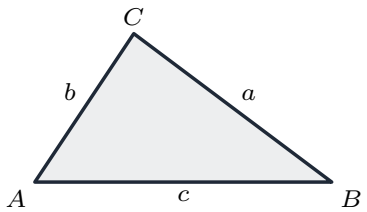
Trigonometry

- An angle can be measured in **degrees** or **radians**; π radians $= 180°$. Check your calculator's mode before every question.
- The **unit circle** extends sine and cosine beyond $90°$; they repeat every $360°$, so they are **periodic**.
- A **bearing** is measured clockwise from north and written with three figures, such as $075°$.
- **Angles of elevation and depression** are measured from the horizontal, up and down. Add the observer's eye height if the question asks for height above the ground.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

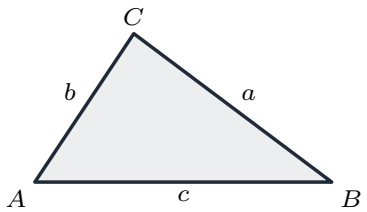
Exercise 2.1

2.1 A right triangle has opposite side 6 cm and hypotenuse 10 cm for an acute angle theta. Find theta to 1 decimal place. [2]



Exercise 2.1

2.1 A right triangle has opposite side 6 cm and hypotenuse 10 cm for an acute angle theta. Find theta to 1 decimal place. [2]



2.1

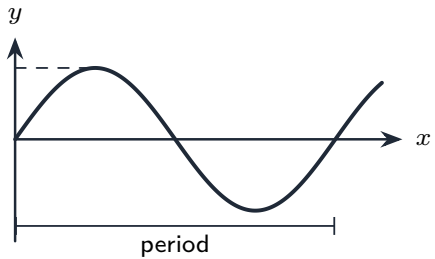
Use sine because opposite and hypotenuse are given.

$$\theta = \sin^{-1}\left(\frac{o}{h}\right) = \sin^{-1}\left(\frac{6}{10}\right) \approx 36.9^\circ$$

Angle 36.9 degrees.

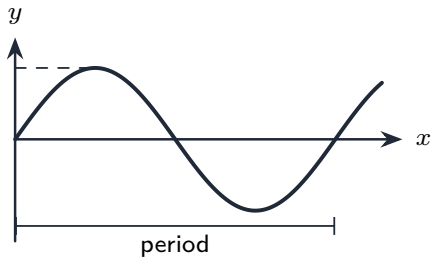
Exercise 2.2

2.2 Convert $150°$ to radians, giving an exact answer in terms of pi. [2]



Exercise 2.2

2.2 Convert $150°$ to radians, giving an exact answer in terms of pi. [2]



2.2

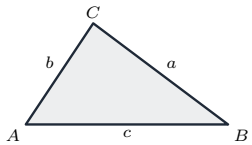
Multiply the degree measure by pi over 180.

$$\theta_{rad} = \theta_{deg}\frac{\pi}{180} = 150\frac{\pi}{180} = \frac{5\pi}{6}$$

Exact answer $5\pi/6$ radians.

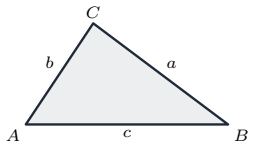
Exercise 2.3

2.3 In triangle ABC, $A = 40°$, $B = 65°$ and side a opposite A is 8 cm. Find side b opposite B to 3 significant figures. [3]



Exercise 2.3

2.3 In triangle ABC, $A = 40°$, $B = 65°$ and side a opposite A is 8 cm. Find side b opposite B to 3 significant figures. [3]



2.3

Pair each side with its opposite angle.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$b = \frac{a \sin B}{\sin A} = \frac{8 \sin 65°}{\sin 40°}$$

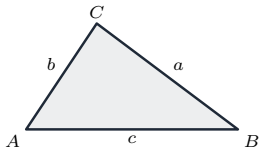
Correct substitution or rearrangement.

$$b = \frac{8 \sin 65°}{\sin 40°} \approx 11.3 \text{ cm}$$

Correct length.

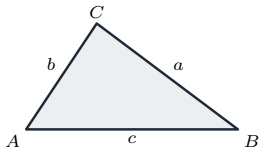
Exercise 2.4

2.4 Two sides of a triangle are 7 and 9 cm with included angle 60°. Find the third side to 3 significant figures. [3]



Exercise 2.4

2.4 Two sides of a triangle are 7 and 9 cm with included angle 60°. Find the third side to 3 significant figures. [3]



2.4

Use cosine rule because the angle lies between the known sides.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c = \sqrt{a^2 + b^2 - 2ab\cos C} = \sqrt{7^2 + 9^2 - 2(7)(9)\cos 60^\circ}$$

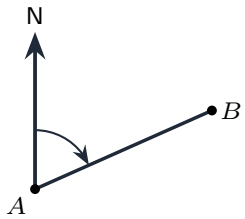
Correct substitution.

$$c = \sqrt{7^2 + 9^2 - 2(7)(9)\cos 60^\circ} = \sqrt{67} \approx 8.19 \text{ cm}$$

Correct third side.

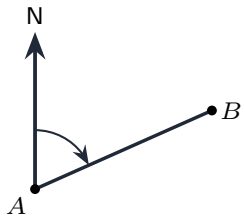
Exercise 3.1

3.1 A walker travels 4 km due east, then 3 km due north. (a) Find the straight-line distance from the start. (b) Find the bearing of the finishing point from the start, to the nearest degree. (c) Find the reverse bearing, also to the nearest degree. [6]



Exercise 3.1

3.1 A walker travels 4 km due east, then 3 km due north. (a) Find the straight-line distance from the start. (b) Find the bearing of the finishing point from the start, to the nearest degree. (c) Find the reverse bearing, also to the nearest degree. [6]



3.1

(a) The east and north components are perpendicular.

$$d = \sqrt{e^2 + n^2} = \sqrt{4^2 + 3^2} = 5 \text{ km}$$

Distance 5 km.

Full solution on next slide

Full solution 3.1

3.1

(a) The east and north components are perpendicular.

$$d = \sqrt{e^2 + n^2} = \sqrt{4^2 + 3^2} = 5 \text{ km}$$

Distance 5 km.

(b) Measure clockwise from north, so east is opposite and north adjacent.

$$\beta = \tan^{-1}\left(\frac{e}{n}\right) = \tan^{-1}\left(\frac{4}{3}\right) \approx 53.13^\circ$$

Bearing 053 degrees.

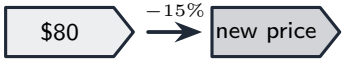
(c) A reverse direction differs by 180 degrees.

$$\beta_{reverse} = \beta + 180^\circ \approx 53.13^\circ + 180^\circ = 233.13^\circ$$

Reverse bearing 233 degrees.

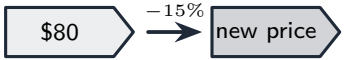
Exercise 3.2

3.2 An observer on level ground measures a tower elevation of 35°. After moving 10 m straight towards the tower, the elevation is 50°. The instrument remains 1.5 m above the ground. Let d be the initial horizontal distance to the tower base and h the tower height above the instrument. (a) Form two equations. (b) Find d and the total tower height to 3 significant figures. (c) Explain why the instrument height is added only once. [8]



Exercise 3.2

3.2 An observer on level ground measures a tower elevation of 35°. After moving 10 m straight towards the tower, the elevation is 50°. The instrument remains 1.5 m above the ground. Let d be the initial horizontal distance to the tower base and h the tower height above the instrument. (a) Form two equations. (b) Find d and the total tower height to 3 significant figures. (c) Explain why the instrument height is added only once. [8]



3.2

The near distance is $d - 10$ m. Both triangles share height h above instrument level.

$$h = d\tan 35^\circ$$

First equation.

Full solution on next slide

Full solution 3.2 (1/2)

3.2

The near distance is $d - 10$ m. Both triangles share height h above instrument level.

$$h = d\tan 35^\circ$$

First equation.

$$h = (d - 10)\tan 50^\circ$$

Second equation. Equate the two expressions for h.

$$d\tan 35^\circ = (d - 10)\tan 50^\circ$$

Rearrange to isolate d.

$$, \qquad 10\tan 50^\circ$$

Full solution 3.2 (2/2)

$$a = \frac{}{\tan 50^\circ - \tan 35^\circ}$$

$$d = \frac{10 \tan 50^\circ}{\tan 50^\circ - \tan 35^\circ} \approx 24.2 \text{ m}$$

Correct initial distance. Use the unrounded distance to find h.

$$h = d \tan 35^\circ \approx 16.9765 \text{ m}$$

Correct height above instrument level.

$$H = h + 1.5 \approx 16.9765 + 1.5 = 18.5 \text{ m} \quad (3 \text{ s.f.})$$

Total height. Both observations use the same instrument level; 1.5 m is one offset from that shared horizontal line to the ground.