Exercise walk-through

1.4
Geometry: Plane, Solid, and Euclidean Geometry

GAC Mathematics

Algebra: graphs and coordinate geometry

- Lines are **parallel** when their gradients are equal, and **perpendicular** when their gradients multiply to -1.
- The **midpoint** of a segment is the average of the endpoints; the **distance** between two points comes from Pythagoras: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- A quadratic graph is a **parabola**. Its **vertex** is the turning point; its **roots** are where it meets the x-axis. The path of a thrown ball is close to a parabola.
- To solve two equations **graphically**, draw both and read where they cross; check the answer algebraically, because a drawing is only approximate.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Geometry: plane, solid and Euclidean geometry

- **Congruent** shapes are identical in size and shape; **similar** shapes have the same shape, with all lengths in one ratio.
- **Pythagoras' theorem**: in a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse**. Ancient Chinese mathematicians knew it as the *gougu* rule; the *Zhoubi Suanjing* shows the 3, 4, 5 triangle.
- **Area** is measured in square units and **volume** in cubic units. A cylinder has volume $\pi r^2 h$; a sphere has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.
- △ Scaling is not linear. Double every length of a solid and its area is multiplied by 4, its volume by 8. Galileo used this in 1638 to explain why a giant animal needs much thicker bones: its weight grows with volume, but bone strength grows only with cross-sectional area.

$$A = \tfrac{1}{2}(3)(4) = 6 \text{ cm}^2 \qquad V = A \times L = 6 \times 10 = 60 \text{ cm}^3$$

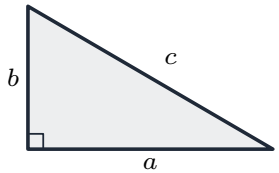
Trigonometry

- An angle can be measured in **degrees** or **radians**; π radians $= 180°$. Check your calculator's mode before every question.
- The **unit circle** extends sine and cosine beyond $90°$; they repeat every $360°$, so they are **periodic**.
- A **bearing** is measured clockwise from north and written with three figures, such as $075°$.
- **Angles of elevation and depression** are measured from the horizontal, up and down. Add the observer's eye height if the question asks for height above the ground.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

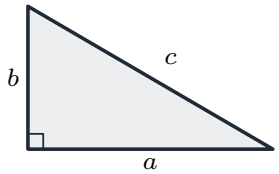
Exercise 2.1

2.1 A right triangle has perpendicular sides 5 and 12 cm. Find its hypotenuse. [2]



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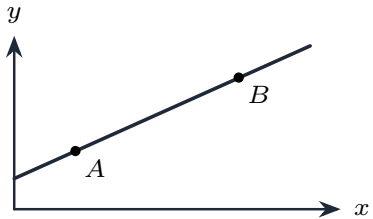
Use Pythagoras because the given sides meet at a right angle.

$$c = \sqrt{a^2 + b^2} = \sqrt{5^2 + 12^2} = 13 \text{ cm}$$

Hypotenuse 13 cm.

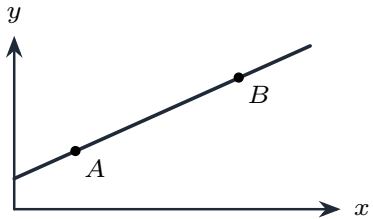
Exercise 2.2

2.2 A triangle has base 8 cm and perpendicular height 6 cm. Find its area. [2]



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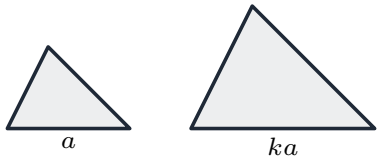
Use half the rectangle area.

$$A = \frac{1}{2}bh = \frac{1}{2}(8)(6) = 24 \text{ cm}^2$$

Area 24 square centimetres.

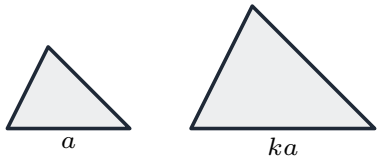
Exercise 2.3

2.3 Two similar triangles have corresponding lengths 4 and 10 cm. The smaller has area 12 cm^2. Find the length scale factor and the larger area. [3]



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2.3

The larger-to-smaller length factor is $k = 10/4 = 2.5$. Areas scale by the square of the length factor.

$$A_L = k^2 A_S = (2.5)^2(12) = 75 \text{ cm}^2$$

Larger area 75 square centimetres.

Exercise 2.4

2.4 A closed cylinder has radius 3 cm and height 8 cm. Find its volume, giving an exact answer in terms of pi and a decimal to 3 significant figures. [3]



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2.4

Use circular area times height.

$$V = \pi r^2 h = \pi(3)^2(8) = 72\pi \text{ cm}^3$$

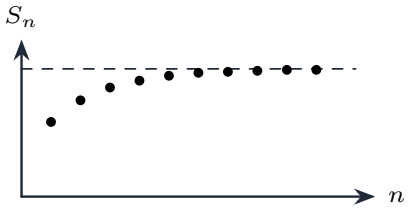
Exact answer.

$$V = 72\pi \text{ cm}^3 \approx 226 \text{ cm}^3$$

Correct 3 significant figures.

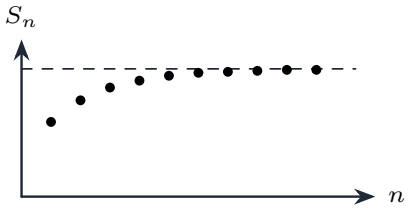
Exercise 3.1

3.1 A rectangular block measures 8 by 5 by 3 cm. (a) Find its volume and total surface area. (b) A geometrically similar block has all lengths doubled. Find its volume and explain why the volume is not merely doubled. [6]



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3.1

Volume uses all three perpendicular lengths.

$$V = lwh = 8(5)(3) = 120 \text{ cm}^3$$

Full solution on next slide

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Full solution 3.1

3.1

Volume uses all three perpendicular lengths.

$$V = lwh = 8(5)(3) = 120 \text{ cm}^3$$

Volume. Sum the three pairs of opposite faces.

$$S = 2(lw + lh + wh)$$

$$S = 2(8(5) + 8(3) + 5(3)) = 158 \text{ cm}^2$$

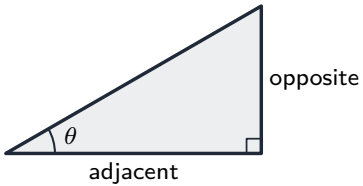
Surface area.

$$V_L = k^3 V_S = 2^3(120) = 960 \text{ cm}^3$$

Larger volume. Each of the three dimensions doubles, so the product grows eightfold.

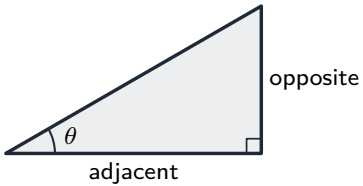
Exercise 3.2

3.2 An open rectangular tank has internal length 80 cm, width 50 cm and depth 40 cm. (a) Find its capacity in litres, using 1000 $\text{cm}^3 = 1$ litre. (b) It initially contains water to depth 25 cm. Find the water volume. (c) A rectangular solid block measuring 20 by 10 by 10 cm is fully submerged without spilling; it does not absorb water. Find the new water depth and state the remaining depth below the rim. [8]



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Capacity is internal rectangular volume.

$$V_T = lwh = 80(50)(40) = 160000 \text{ cm}^3$$

Full solution on next slide

Full solution 3.2 (1/2)

3.2

Capacity is internal rectangular volume.

$$V_T = lwh = 80(50)(40) = 160000 \text{ cm}^3$$

$$C = \frac{V_T}{1000} = \frac{160000}{1000} = 160 \text{ litres}$$

Capacity 160 litres.

$$V_W = lwh_W = 80(50)(25) = 100000 \text{ cm}^3 = 100 \text{ litres}$$

Initial water volume 100 litres. The submerged block displaces its own volume.

$$V_B = l_B w_B h_B = 20(10)(10) = 2000 \text{ cm}^3$$

Full solution 3.2 (2/2)

Block volume.

$$\Delta h = \frac{V_B}{lw} = \frac{2000}{80(50)} = 0.5 \text{ cm}$$

Correct rise.

$$h_{new} = h_W + \Delta h = 25 + 0.5 = 25.5 \text{ cm}$$

New depth.

$$h_{gap} = h_T - h_{new} = 40 - 25.5 = 14.5 \text{ cm}$$

Remaining gap 14.5 cm. The initial depth already exceeds the block's height, so complete submersion is possible.