Exercise walk-through

1.2
Algebra I: Introductory Algebra

GAC Mathematics

Algebra I: introductory algebra

- To solve a **linear equation** , do the same operation to both sides until the letter stands alone.
- A **quadratic equation** has the form $ax^2 + bx + c = 0$ with $a \neq 0$. Factorise when you can, or use $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- **Simultaneous equations** are two equations in two unknowns, solved by substitution or elimination. Two straight lines meet at one point, or never (parallel), or everywhere (the same line).
- △ An inequality **reverses** when you multiply or divide both sides by a negative number: from $-2x > 6$ it follows that $x < -3$.

$$(x+3)(x-5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15$$

Algebra: graphs and coordinate geometry

- Lines are **parallel** when their gradients are equal, and **perpendicular** when their gradients multiply to -1.
- The **midpoint** of a segment is the average of the endpoints; the **distance** between two points comes from Pythagoras: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- A quadratic graph is a **parabola**. Its **vertex** is the turning point; its **roots** are where it meets the x-axis. The path of a thrown ball is close to a parabola.
- To solve two equations **graphically**, draw both and read where they cross; check the answer algebraically, because a drawing is only approximate.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

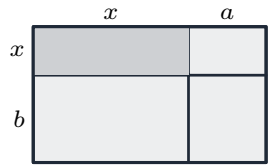
Trigonometry

- An angle can be measured in **degrees** or **radians**; π radians $= 180°$. Check your calculator's mode before every question.
- The **unit circle** extends sine and cosine beyond $90°$; they repeat every $360°$, so they are **periodic**.
- A **bearing** is measured clockwise from north and written with three figures, such as $075°$.
- **Angles of elevation and depression** are measured from the horizontal, up and down. Add the observer's eye height if the question asks for height above the ground.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

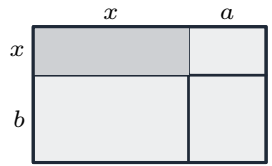
Exercise 2.1

2.1 Expand and simplify $3(2x - 5) + 4x$. [2]



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2.1

Distribute 3 to both terms.

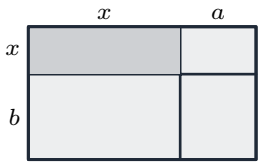
$$E = 3(2x - 5) + 4x = 6x - 15 + 4x$$

$$E = 10x - 15$$

Correct simplified expression.

Exercise 2.2

2.2 Solve $5x - 7 = 18$. [2]



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2.2

Add 7 to both sides, then divide by 5.

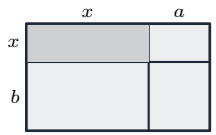
$$5x = 18 + 7 = 25$$

$$x = \frac{25}{5} = 5$$

Correct solution.

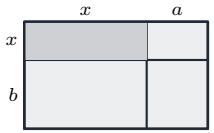
Exercise 2.3

2.3 Factorise $x^2 - x - 12$, then solve $x^2 - x - 12 = 0$. [3]



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2.3

Two numbers with product negative 12 and sum negative 1 are negative 4 and 3.

$$x^2 - x - 12 = (x - 4)(x + 3)$$

Correct factorisation. A zero product means at least one factor is zero.

$$x - 4 = 0 \text{ or } x + 3 = 0$$

$$x = 4 \text{ or } x = -3$$

Both roots.

Exercise 2.4

2.4 Solve $4 - 2x \leq 10$. State whether the boundary value is included. [3]

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2.4

Subtract 4, then divide by negative 2 and reverse the sign.

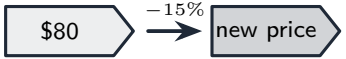
$$-2x \leq 10 - 4 = 6$$

$$x \geq \frac{6}{-2} = -3$$

Correct inequality. Negative 3 is included because the original sign allows equality.

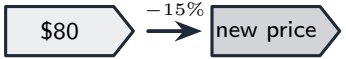
Exercise 3.1

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3.1

Let a and s be the adult and student prices. Translate both totals.

$$3a + 2s = 46, \qquad 2a + 4s = 44$$

Double the first equation so the student coefficients match.

$$6a + 4s = 92$$

Full solution on next slide

Full solution 3.1 (1/2)

3.1

Let a and s be the adult and student prices. Translate both totals.

$$3a + 2s = 46, \qquad 2a + 4s = 44$$

Double the first equation so the student coefficients match.

$$6a + 4s = 92$$

Subtract the second equation.

Full solution 3.1 (2/2)

$$4a = 92 - 44 = 48$$

$$a = \frac{48}{4} = 12$$

Adult price 12.

$$s = \frac{46 - 3a}{2} = \frac{46 - 3(12)}{2} = 5$$

Student price 5. Checks: $3(12) + 2(5) = 46$ and $2(12) + 4(5) = 44$. Both checks earn B1.

Exercise 3.2

3.2 A rectangle has width x metres and length $(x+3)$ metres. Its area is 40 m^2. (a) Form and solve an equation for x. (b) Choose the physical solution and find the perimeter. (c) A second rectangle has the same width x but length $(x+5)$ metres. Find the increase in area. [8]



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3.2

Area equals length times width.

$$A = lw$$

$$40 = x(x+3) \quad \Longrightarrow \quad x^2 + 3x - 40 = 0$$

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Full solution 3.2 (1/2)

3.2

Area equals length times width.

$$A = lw$$

$$40 = x(x+3) \quad \Longrightarrow \quad x^2 + 3x - 40 = 0$$

Correct equation. Factorise.

$$x^2 + 3x - 40 = (x+8)(x-5)$$

$$x = -8 \text{ or } x = 5$$

Both roots. Choose $x = 5$ because a physical width must be positive.

Full solution 3.2 (2/2)

$$P = 2(l + w) = 2((5 + 3) + 5) = 26 \text{ m}$$

Perimeter 26 m.

$$\Delta A = A_2 - A_1 = x(x + 5) - x(x + 3)$$

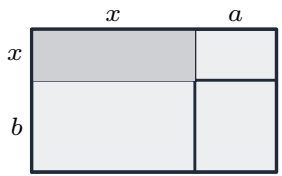
Valid comparison.

$$\Delta A = 5(5 + 5) - 5(5 + 3) = 50 - 40 = 10 \text{ m}^2$$

Increase 10 square metres.

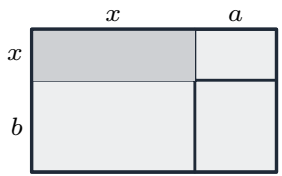
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3.3 Solve $2x + y = 13$ and $x - y = 2$, showing elimination. Check your pair in both original equations. [4]



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3.3

Adding eliminates y: $3x = 15$; $x = 5$; substitution gives $y = 3$; checks $2(5) + 3 = 13$ and $5 - 3 = 2$.