Exercise walk-through

1.1 Terminology and Arithmetic Review

GAC Mathematics

Algebra I: introductory algebra

- To solve a **linear equation** , do the same operation to both sides until the letter stands alone.
- A **quadratic equation** has the form $ax^2 + bx + c = 0$ with $a \neq 0$. Factorise when you can, or use $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- **Simultaneous equations** are two equations in two unknowns, solved by substitution or elimination. Two straight lines meet at one point, or never (parallel), or everywhere (the same line).
- △ An inequality **reverses** when you multiply or divide both sides by a negative number: from $-2x > 6$ it follows that $x < -3$.

$$(x+3)(x-5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15$$

Algebra: graphs and coordinate geometry

- Lines are **parallel** when their gradients are equal, and **perpendicular** when their gradients multiply to -1.
- The **midpoint** of a segment is the average of the endpoints; the **distance** between two points comes from Pythagoras: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- A quadratic graph is a **parabola**. Its **vertex** is the turning point; its **roots** are where it meets the x-axis. The path of a thrown ball is close to a parabola.
- To solve two equations **graphically**, draw both and read where they cross; check the answer algebraically, because a drawing is only approximate.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Trigonometry

- An angle can be measured in **degrees** or **radians**; π radians $= 180°$. Check your calculator's mode before every question.
- The **unit circle** extends sine and cosine beyond $90°$; they repeat every $360°$, so they are **periodic**.
- A **bearing** is measured clockwise from north and written with three figures, such as $075°$.
- **Angles of elevation and depression** are measured from the horizontal, up and down. Add the observer's eye height if the question asks for height above the ground.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Exercise 2.1

2.1 Evaluate $18 - 3(4 - 7)$, showing the bracket step. [2]

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2.1

Known: a bracket is multiplied before the subtraction.

$$V = 18 - 3(4 - 7) = 18 - 3(-3)$$

$$V = 18 - (-9) = 27$$

Final result 27.

Exercise 2.2

2.2 Calculate $\frac{3}{4} - \frac{2}{5}$ in simplest form. [2]



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2.2

Use a common denominator of 20.

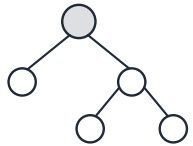
$$F = \frac{3}{4} - \frac{2}{5} = \frac{15}{20} - \frac{8}{20}$$

$$F = \frac{15 - 8}{20} = \frac{7}{20}$$

Final fraction 7/20.

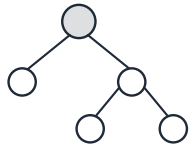
Exercise 2.3

2.3 Find the highest common factor and lowest common multiple of 18 and 24. [3]



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2.3

Prime factors give the common and combined powers.

$$18 = 2 \times 3^2, \qquad 24 = 2^3 \times 3$$

$$\text{HCF} = 2 \times 3 = 6$$

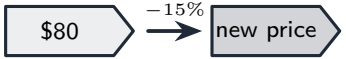
HCF 6.

$$\text{LCM} = 2^3 \times 3^2 = 72$$

LCM 72.

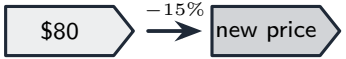
Exercise 2.4

2.4 A price rises from 120 to 150 units. Calculate the percentage increase and the percentage decrease needed to return from 150 to 120. [3]



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2.4

The change is 30, but the two bases differ.

$$r_{up} = \frac{P_{new} - P_{old}}{P_{old}} \times 100\%$$

$$r_{up} = \frac{150 - 120}{120} \times 100\% = 25\%$$

Increase 25%.

Full solution on next slide

Full solution 2.4

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Increase 25%.

$$r_{down} = \frac{P_{old} - P_{new}}{P_{old}} \times 100\%$$

$$r_{down} = \frac{150 - 120}{150} \times 100\% = 20\%$$

Required decrease 20%.

Exercise 3.1

3.1 Two school bells ring every 12 and 18 minutes. They ring together at 09:00. (a) Find when they next ring together. (b) A supplier offers 48 pens for 36 units or 60 pens for 42 units. Compare unit prices and state which has the lower cost per pen. [6]



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3.1

(a) Simultaneous ringing repeats after the LCM.

$$12 = 2^2 \times 3, \qquad 18 = 2 \times 3^2$$

$$t = \text{LCM}(12, 18) = 2^2 \times 3^2 = 36 \text{ min}$$

36 minutes. Next time 09:36.

Full solution on next slide

Full solution 3.1

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36 minutes. Next time 09:36.

(b) Divide cost by pen count.

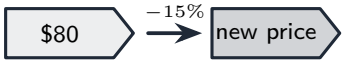
$$u = \frac{C}{n}$$

$$u_A = \frac{36}{48} = 0.75, \qquad u_B = \frac{42}{60} = 0.70$$

Both unit prices correct. The 60-pen offer is lower per pen; its total outlay is nevertheless higher.

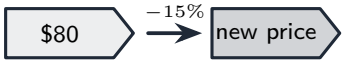
Exercise 3.2

3.2 A club sells an item for 72 units after a 10% discount on its marked price. (a) Find the marked price. (b) A different offer takes 15% off the same marked price and then adds a 4-unit delivery charge. Find its total. (c) Compare the offers if the first has no delivery charge. Explain why adding 10% to 72 does not recover the marked price. [8]



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3.2

(a) Discount multiplier is 0.90.

$$P_{sale} = 0.90 P_{marked}$$

$$P_{marked} = \frac{P_{sale}}{0.90}$$

Full solution on next slide

Full solution 3.2 (1/2)

3.2

(a) Discount multiplier is 0.90.

$$P_{sale} = 0.90 P_{marked}$$

$$P_{marked} = \frac{P_{sale}}{0.90}$$

$$P_{marked} = \frac{72}{0.90} = 80$$

Marked price 80.

Full solution 3.2 (2/2)

(b) Apply the 15% multiplier before adding delivery.

$$C = 0.85P_{marked} + D$$

$$C = 0.85(80) + 4 = 68 + 4 = 72$$

Discounted item price 68. Delivered total 72.

(c) Totals are equal under the stated delivery assumptions. Adding 10% to the sale price uses 72 rather than the original base 80.

$$P' = 72(1 + 10/100) = 79.2 \neq 80$$

The numerical check or a clear inverse-multiplier explanation earns the final point.