Numbers and the number system

4MA1 full topic 1 package: Foundation 1.1-1.11 and Higher additions with June 2024 demand

Numbers and the number system

- use the order of operations, negative numbers, factors, multiples and primes;
- calculate with fractions, decimals and percentages;
- simplify surds and work with sets and Venn diagrams;

Numbers and the number system (2)

- use **ratio** and proportion;
- round, estimate, find bounds and use **standard form**.

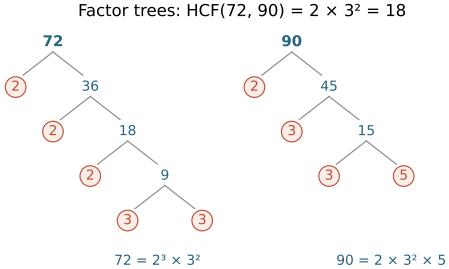
Integers, four rules and the hierarchy of operations

- Adding a negative number is the same as subtracting: $5 + (-3) = 2$.
- Subtracting a negative number is the same as adding: $5 - (-3) = 8$.
- Multiplying or dividing two numbers with the same sign gives a positive answer, and with different signs a negative answer: $(-4) \times (-3) = 12$, $(-12) \div 3 = -4$.

Integers, four rules and the hierarchy of operations (2)

- A **factor** tree gives $72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$.
- The **highest common factor 最大公因数** (HCF) uses each prime in both numbers, with the lower power: $2 \times 3^2 = 18$.
- The **lowest common multiple 最小公倍数** (LCM) uses every prime, with the higher power: $2^3 \times 3^2 \times 5 = 360$.

Figure: Integers, four rules and the hierarchy of operations



- **Factor** trees: $72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$, so HCF $= 2 \times 3^2 = 18$
- Match each part of the diagram to the text.

Fractions

- To add or subtract, use a **common denominator 公分母**: $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$.
- To multiply, multiply the tops and multiply the bottoms: $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.
- To divide, multiply by the reciprocal of the second fraction: $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$.

Fractions (2)

- To find a fraction of an amount, multiply: $\frac{3}{4}$ of 48 is $48 \div 4 \times 3 = 36$.
- Let $x = 0.306306\ldots$. Three digits repeat, so multiply by 1000: $1000x = 306.306306\ldots$
- Subtract: $999x = 306$, so $x = \frac{306}{999} = \frac{34}{111}$ (dividing by 9).

Decimals, indices, factors and multiples

- multiply by adding the indices: $2^3 \times 2^4 = 2^7$;
- divide by subtracting the indices: $2^7 \div 2^3 = 2^4$.
- $360 = 2^3 \times 3^2 \times 5$ and $84 = 2^2 \times 3 \times 7$.
- HCF $= 2^2 \times 3 = 12$; LCM $= 2^3 \times 3^2 \times 5 \times 7 = 2520$.

Surds

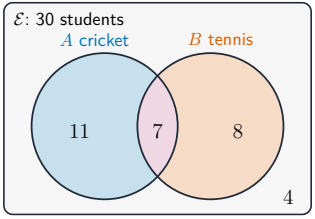
- Simplify using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ with a **square number**: $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$.
- You can add only like surds: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$, but $\sqrt{2} + \sqrt{3}$ cannot be simplified.
- for $\frac{a}{\sqrt{b}}$, multiply the top and bottom by $\sqrt{b}$: $\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$;
- for $\frac{a}{c-\sqrt{b}}$, multiply the top and bottom by $c + \sqrt{b}$, because $(c - \sqrt{b})(c + \sqrt{b}) = c^2 - b$.

$$\frac{5}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{5(3+\sqrt{2})}{9-2} = \frac{5(3+\sqrt{2})}{7}$$

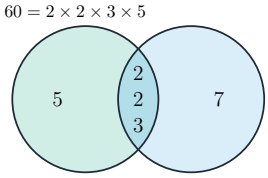
Sets

- $30 - 4 = 26$ play at least one sport, so $n(A \cup B) = 26$.
- $n(A) + n(B) = 33$, which counts the students who play both twice. So $n(A \cap B) = 33 - 26 = 7$.
- Then 11 play cricket only and 8 play tennis only, and $n(A') = 12$.

Figure: Sets



$n(A) = 11 + 7 = 18$ $n(A \cup B) = 11 + 7 + 8 = 26$
$n(A \cap B) = 7$ $n(A') = 8 + 4 = 12$



HCF = overlap $= 2 \times 2 \times 3 = 12$
LCM = every region $= 5 \times 2 \times 2 \times 3 \times 7 = 420$

count each region once; HCF multiplies the overlap, LCM multiplies every region

- Left: the class of worked example 5. Right: the prime factors of 60 and 84, with 2, 2, 3 in the overlap, so the HCF is 12 and the LCM is 420.
- Match each part of the diagram to the text.

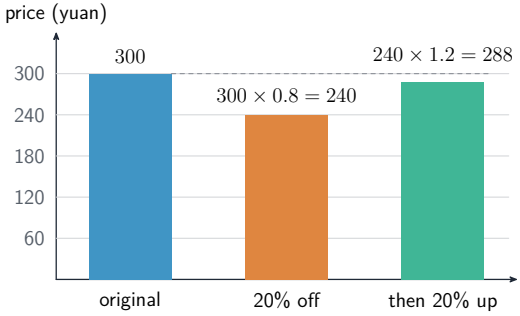
Percentages

- an increase of 12% multiplies by 1.12;
- a decrease of 20% multiplies by 0.8;
- to find the original value after a change, divide by the **multiplier** (a reverse percentage).

Percentages (2)

- The **multiplier** is $1 - 0.17 = 0.83$, so the original price is $6225 \div 0.83 = 7500$ rupees.
- Check: $7500 \times 0.83 = 6225$.
- Investment: $2000 \times 1.03^4 = £2251.02$.
- Car: $12\,000 \times 0.85^3 = £7369.50$.

Figure: Percentages



- 20% off then 20% up gives 288, not 300.
- The second 20% is of 240.

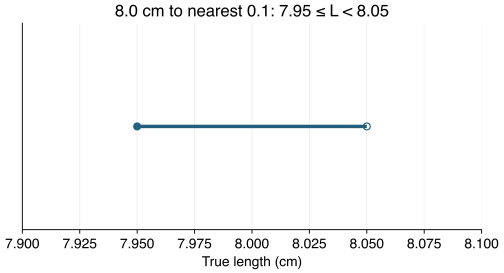
Ratio and proportion

- There are $2 + 5 = 7$ parts, so one part is $350 \div 7 = £50$.
- The shares are $2 \times 50 = £100$ and $5 \times 50 = £250$. Check: $100 + 250 = 350$.

Rounding, bounds, estimation and standard form

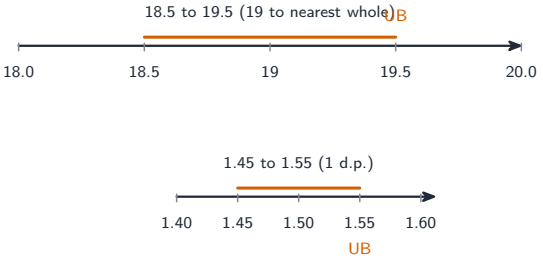
- **Upper bound** of the speed: 19.5 km/h. Upper bound of the time: 1.55 h.
- Distance = speed $\times$ time, so the largest possible distance is $19.5 \times 1.55 = 30.2$ km (3 s.f.).

Figure: Rounding, bounds, estimation and standard form



- 8.0 cm to the nearest 0.1 cm: $7.95 \leq L < 8.05$
- Match each part of the diagram to the text.

Figure: Rounding, bounds, estimation and standard form (2)



maximise distance: UB speed $\times$ UB time
$= \ 19.5 \times 1.55 \ = \ 30.2$ km (3 s.f.)

- Upper bounds: 19.5 and 1.55.
- Largest distance: $19.5 \times 1.55 = 30.2$ km.

Number in context: units, time and calculators

- Length: 1 km $= 1000$ m, 1 m $= 100$ cm, 1 cm $= 10$ mm.
- Mass: 1 kg $= 1000$ g. Capacity: 1 litre $= 1000$ cm^3.
- Area and volume: 1 m$^2 = 100^2 = 10\,000$ cm^2 and 1 m$^3 = 1\,000\,000$ cm^3.
- Time: 1 hour $= 60$ minutes, so 2.25 hours is 2 hours 15 minutes, not 2 hours 25 minutes.

Sources for this lesson

- Pearson Edexcel, *International GCSE (9–1) Mathematics A specification* (Issue 2, 2017), topic 1.
- Pearson Edexcel, 4MA1 Papers 1H and 2H, June 2024 (adapted).
- K. Plofker, *Mathematics in India* (Princeton University Press, 2009), chapter 5, on Brahmagupta.

Common errors

- Working from left to right instead of using BIDMAS.
- Adding fractions by adding the tops and adding the bottoms.
- Finding a reverse percentage by taking the percentage off the new value: the original price in worked example 6 is not 6225×1.17.

Common errors (2)

- Using simple interest when the question says compound.
- Thinking that $\sqrt{2} + \sqrt{3} = \sqrt{5}$.
- Giving the **upper bound** of 19 (to the nearest whole) as 19.4 or 19.49; it is 19.5.
- Writing 15×10^{10} as the final answer; in **standard form** the first number must be less than 10.

Key terms

- **Prime number**: a number with exactly two factors, 1 and itself.
- **Highest common factor**: the largest number that divides into two or more numbers.
- **Surd**: a root that cannot be written as an exact fraction.

Key terms (2)

- **Intersection**: the elements that are in both of two sets.
- **Multiplier**: the number you multiply by to make a percentage change.
- **Upper bound**: the largest value that rounds to a given measurement.

Check yourself

- 1.
- Work out $8 - 2 \times (3 - 5)^2$. 2.
- Work out $2\frac{1}{2} \div \frac{3}{4}$. 3.
- Simplify $\sqrt{72}$. 4.