Course Companion

Pearson Edexcel · International GCSE · Mathematics A

Every topic handout in one volume

全部专题讲义合集

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Numbers and the number system

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4MA1 topic 1 (specification Issue 2, November 2017) is the number heart of the qualification: Foundation rows 1.1–1.11 run from integers and the four rules through fractions, decimals, indices, sets, percentages, ratio, bounds and standard form to everyday number work; Higher adds recurring-decimal conversion, surds, algebraic sets, repeated percentage change and bound problems. On the June 2024 papers this strand supplied reverse percentage (2HR Q8), standard form (2HR Q9), the recurring-decimal proof (2HR Q18), upper-bound multiplication (2HR Q19), surds rationalisation (1HR Q17 family) and prime factors with Venn regions (1HR Q11 family).

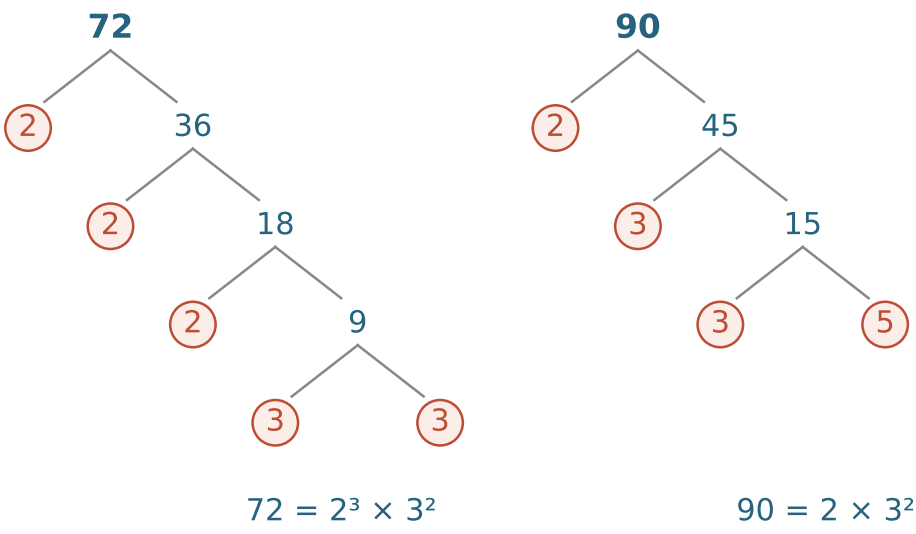
Must-know for topic 1:

Family	Formulae
percentages	multiplier $1+\frac{r}{100}$ (increase), $1-\frac{r}{100}$ (decrease); reverse: original = final $\div$ multiplier
bounds	value v to 1 d.p. has LB $= v-0.05$, UB $= v+0.05$; maximise products by multiplying upper bounds
standard form	$a\times 10^n$ with $1\le a<10$; indices add on multiplication, subtract on division
ratio	share by $\dfrac{\text{parts owned}}{\text{total parts}}\times$ amount

Integers, four rules and the hierarchy of operations

Integers cover positive, negative and zero; place value names each digit's worth. Directed numbers appear in practical settings (temperature, debt, sea level); ordering puts them on the number line. The four rules with negatives: same signs add, different signs subtract, two negatives make a positive. **Brackets, then indices, then divide/multiply, then add/subtract** (BIDMAS): $3+4\times 2^2=3+16=19$, while $(3+4)\times 2=14$. Odd, even, **prime numbers** 质数 (exactly two factors), **factors** 因数 and **multiples** 倍数 classify whole numbers; common factors and common multiples lead to the HCF and LCM.

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Factor trees: $72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$, so HCF $= 2 \times 3^2 = 18$

Fractions

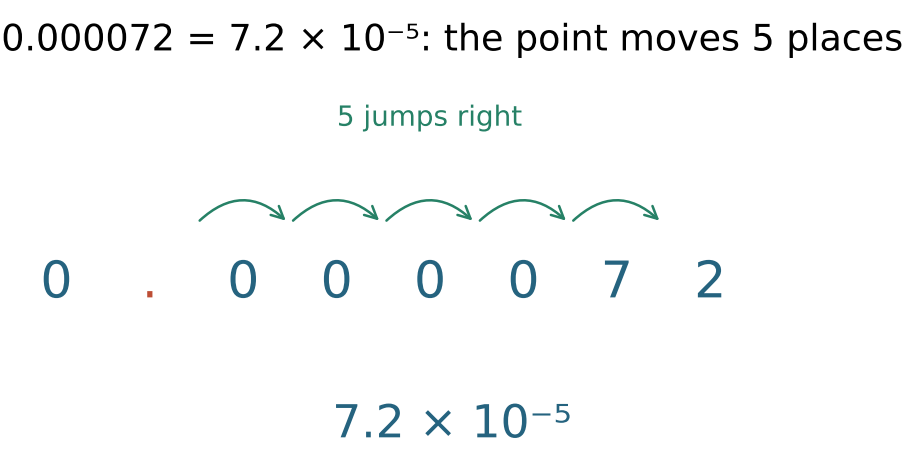
Equivalent fractions 等值分数 cancel or extend: $\frac{8}{12} = \frac{2}{3}$. **Mixed numbers** 带分数 and **vulgar fractions** convert by division with remainder. A **common denominator** 公分母 lets fractions be ordered, compared, added and subtracted: $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$. A fraction OF a quantity multiplies: $\frac{3}{4}$ of 48 is 36; expressing one number as a fraction of another divides (7 as a fraction of 28 is $\frac{1}{4}$). Multiplying fractions multiplies numerators and denominators; dividing multiplies by the reciprocal – unit fractions as multiplicative inverses ($\frac{1}{3}$ undoes 3). Fractions convert to decimals (divide) and percentages (multiply by 100).

Recurring decimals 循环小数 (Higher) convert by algebra: let $x = 0.306306\ldots$; then $1000x = 306.306\ldots$; subtracting, $999x = 306$, so $x = \frac{306}{999} = \frac{34}{111}$ (June 2024 2HR Q18). A terminating decimal is a fraction with denominator a power of ten.

Decimals, indices, factors and multiples

Decimal place value runs tenths, hundredths, thousandths; ordering compares the highest-value digits first. **Square numbers** 平方数 $(1, 4, 9, \ldots)$, **cube numbers** 立方数 $(1, 8, 27, \ldots)$, their roots, and **index notation** 指数 with the multiplication and division laws for positive integer powers: $2^3 \times 2^4 = 2^7$ and $2^7 \div 2^3 = 2^4$. Every integer factors into primes: $360 = 2^3 \times 3^2 \times 5$; June 2024 1HR Q11 family factors large numbers into primes and reads products of regions from a Venn diagram ($19600 = 2^4 \times 5^2 \times 7^2$). The **HCF** 最大公因数 takes the lowest power of each common prime; the **LCM** 最小公倍数 takes the highest power of every prime: for $360 = 2^3 \cdot 3^2 \cdot 5$ and $84 = 2^2 \cdot 3 \cdot 7$,

HCF $= 2^2 \cdot 3 = 12$ and LCM $= 2^3 \cdot 3^2 \cdot 5 \cdot 7 = 2520$.



Moving the point 5 places right gives 7.2, so $0.000072 = 7.2 \times 10^{-5}$

Surds

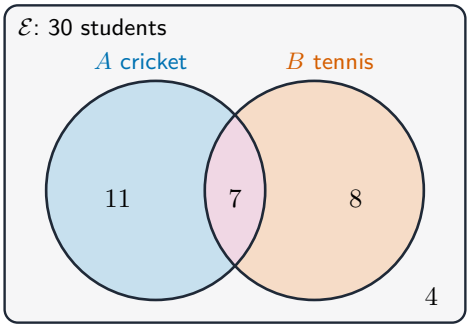
A **surd** 根式 is an exact irrational root such as $\sqrt{2}$. Simplify by extracting squares: $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$, using $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$. Adding needs like surds: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$, but $\sqrt{2} + \sqrt{3}$ does not simplify.

Rationalising the denominator 分母有理化 removes the root by multiplying top and bottom by the conjugate – a difference of two squares clears it exactly. Worked case: $\dfrac{5}{3-\sqrt{2}} = \dfrac{5(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \dfrac{5(3+\sqrt{2})}{9-2} = \dfrac{5(3+\sqrt{2})}{7}$; and $\dfrac{6}{\sqrt{3}} = \dfrac{6\sqrt{3}}{3} = 2\sqrt{3}$. June 2024 1HR Q17 family rationalises a denominator built from $2-\sqrt{3}$ to an answer of the family $\frac{7(2+\sqrt{3})}{4}$ – since $(2-\sqrt{3})(2+\sqrt{3}) = 1$, the $(2+\sqrt{3})$ multiplier clears the surd exactly.

Sets

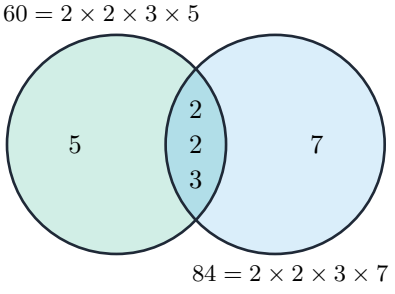
A **set** 集合 is a collection of elements: $A = \{2, 4, 6, 8\}$. Notation: $\cup$ (**union** 并集, in either set), $\cap$ (**intersection** 交集, in both), $\in$ and $\notin$ (**membership** 属于). The **universal set** 全集 $\mathcal{E}$ contains everything under discussion; the **empty set** 空集 $\emptyset$ has no elements; the **complement** 补集 A' holds everything in $\mathcal{E}$ outside A. **Venn diagrams** 维恩图 draw the regions – at Higher with algebraic sets, **subsets** 子集 ($B \subset A$: every element of B lies in A), the element count $n(A)$ 元素个数, and practical uses (polls, prime-factor regions). June 2024 1HR Q11 family places the

prime factors of two numbers in a two-set Venn and reads products such as AB, A^2 and B^2 from combined regions – the HCF is the overlap's product, the LCM everything's product.



$n(A) = 11 + 7 = 18$ $\quad n(A \cup B) = 11 + 7 + 8 = 26$

$n(A \cap B) = 7$ $\quad n(A') = 8 + 4 = 12$



HCF = overlap $= 2 \times 2 \times 3 = 12$

LCM = every region $= 5 \times 2 \times 2 \times 3 \times 7 = 420$

count each region once; HCF multiplies the overlap, LCM multiplies every region

Left: a Venn diagram of 30 students, 11 cricket only, 7 both, 8 tennis only and 4 neither, with $n(A) = 18$, $n(A \cap B) = 7$, $n(A \cup B) = 26$ and $n(A') = 12$ worked out. Right: the prime factors of 60 and 84 in a Venn diagram, with 2, 2, 3 in the overlap, so the HCF is 12 and the LCM is 420.

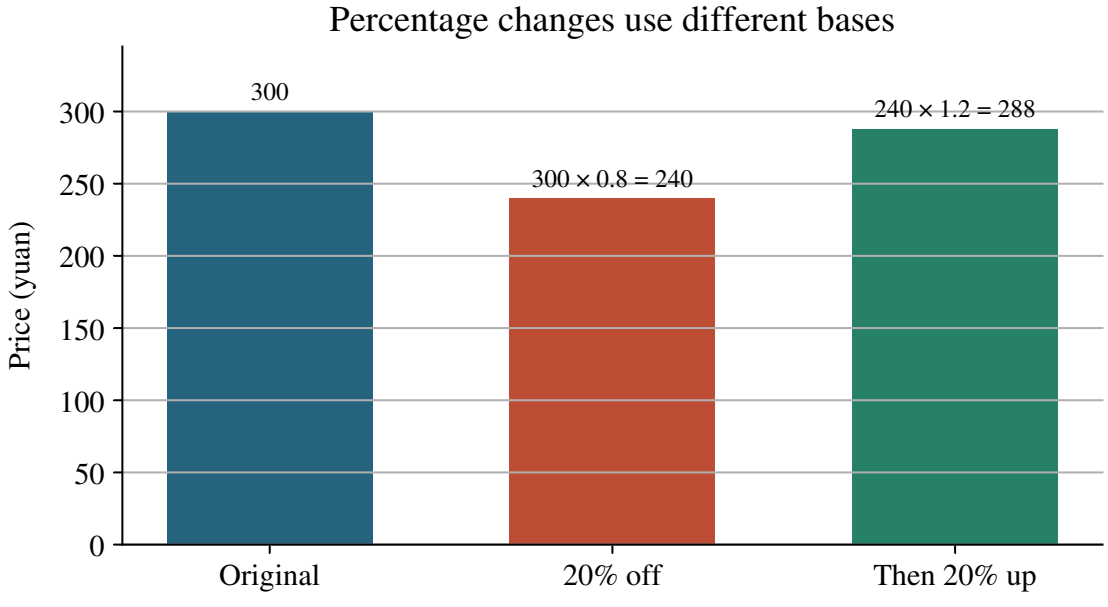
Percentages

Percentage means parts per 100: $35\% = 0.35 = \frac{7}{20}$. Percentages act as **multipliers** 乘数: increase by 12% means $\times 1.12$; decrease by 20% means $\times 0.8$. Reverse percentage divides the final value by the multiplier – June 2024 2HR Q8: a fridge priced at 6225 rupees after a 17% reduction originally cost $\frac{6225}{0.83} = 7500$ rupees. **Compound interest** 复利 and depreciation apply the multiplier repeatedly: $A = P\left(1 \pm \frac{r}{100}\right)^n$ – £2000 at 3% compound for 4 years grows to $2000 \times 1.03^4 =$ £2251.02; a car worth £12000 depreciating 15% per year for 3 years falls to $12000 \times 0.85^3 =$ £7369.50.

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Shop discounts are percentage changes: a 20% reduction multiplies the price by 0.8



20% off then 20% up gives 288, not 300: the second 20% is of 240

Ratio and proportion

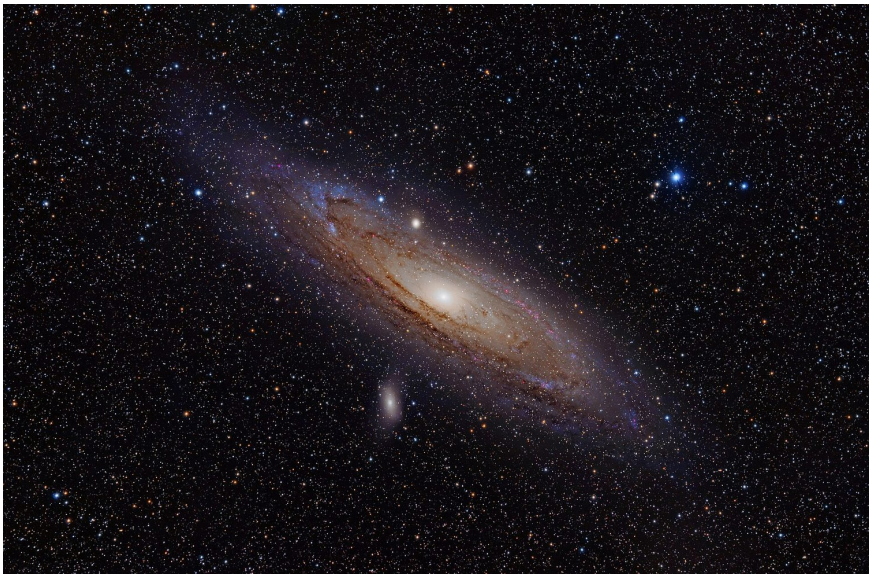
Ratio 比 compares parts: simplify by dividing through ($45 : 60 = 3 : 4$); share by $\frac{\text{parts}}{\text{total}} \times \text{amount}$ – £350 shared in the ratio $2 : 5$ gives $\frac{2}{7} \times 350 = £100$ and $\frac{5}{7} \times 350 = £250$. **Proportion** scales within or between quantities: the unitary method finds one unit first (7 pens cost £4.20, so one pen costs £0.60 and 12 pens cost £7.20); quantities in **direct proportion** 正比例 scale the same way ($y = kx$). Word problems mix

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ratio with fractions and percentages – keep the units explicit and check the answer is sensible.

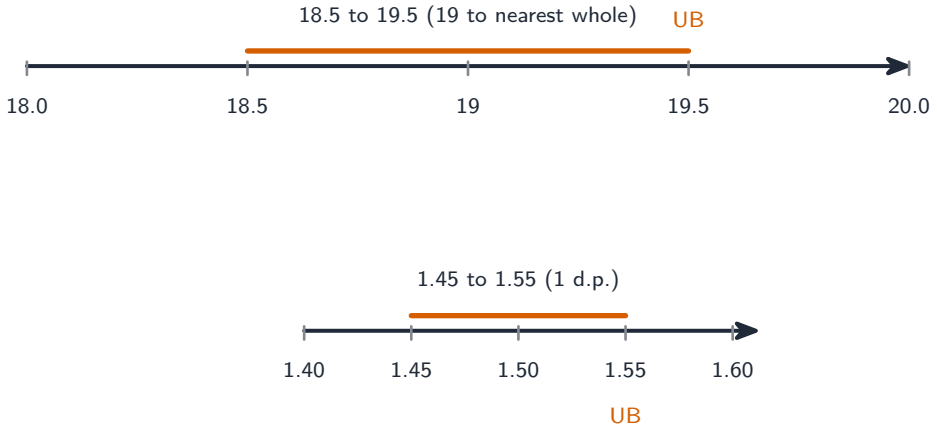
Rounding, bounds, estimation and standard form

Round integers to powers of ten and decimals to significant figures or decimal places: $3.4617 \to 3.5$ (2 s.f.) and 3.46 (2 d.p.). An accurate value carries bounds – 19 to the nearest whole means $18.5 \le v < 19.5$ – the **upper bound** 上界 19.5 and **lower bound** 下界 18.5. Estimation rounds every number to 1 s.f. first: $\frac{4.12\times29.8}{0.187} \approx \frac{4\times30}{0.2} = 600$. Bound problems (Higher) choose the extremes the question needs – June 2024 2HR Q19: distance = speed × time with speed 19 km/h to the nearest whole (so UB 19.5) and time 1.5 h to 1 d.p. (so UB 1.55): the upper bound of the distance is $19.5 \times 1.55 = 30.2$ km (3 s.f.).



A galaxy: distances this large are written in standard form, A × 10^n

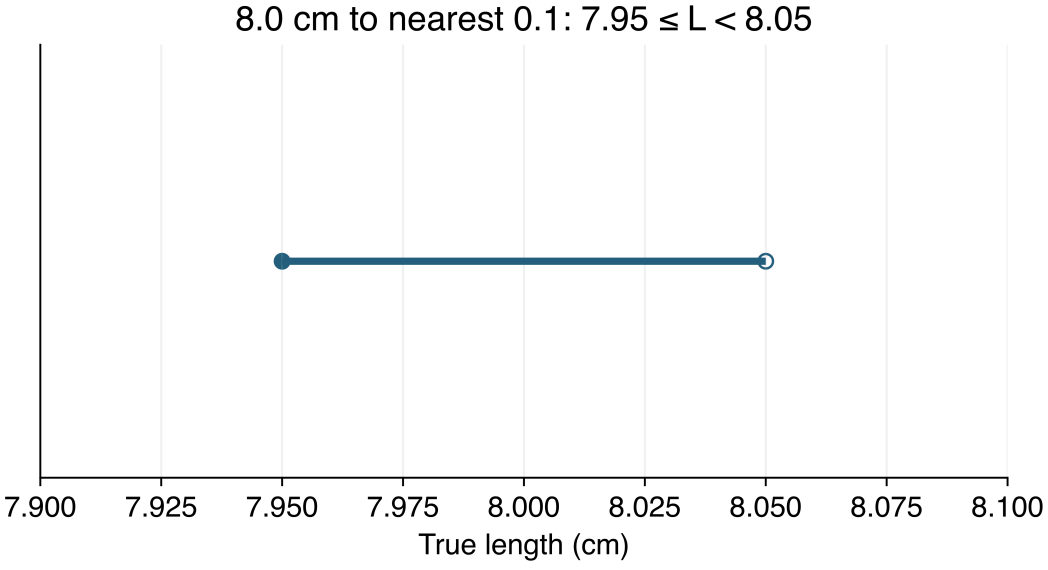
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maximise distance: UB speed $\times$ UB time
$= \ 19.5 \times 1.55 \ = \ 30.2$ km (3 s.f.)

Bounds bar for 19 to the nearest whole (18.5 to 19.5) and 1.5 to 1 d.p. (1.45 to 1.55), with the upper-bound multiplication 19.5 x 1.55 = 30.2 marked.

Standard form 科学记数法 writes $a \times 10^n$ with $1 \le a < 10$: $604000 = 6.04 \times 10^5$ and $0.00007 = 7 \times 10^{-5}$ (June 2024 2HR Q9). Indices add and subtract across the multiplication and division of standard-form numbers, and the calculator's $\times 10^{\wedge}$ key keeps the working exact.



8.0 cm to the nearest 0.1 cm: $7.95 \le L < 8.05$

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Number in context: units, time and calculators

Everyday number – money, timetables, readings – uses the same arithmetic with units attached. Standard units: mass (kg, g), length (km, m, cm, mm), area, volume, time (seconds to years, 12- and 24-hour clocks), and the conversions between them ($1\ m^2 = 10^4\ cm^2$; 1 litre = $1000\ cm^3$). A scientific calculator applies the order of operations correctly – but rounding early loses accuracy: keep full precision through the working and round only the final answer, to the accuracy asked for.

Checklist

- Integers, ordering, directed numbers, four rules, BIDMAS, odd, even, prime, factors and multiples (F 1.1 A–H).
- Equivalent fractions, mixed numbers, common denominators, fraction of a quantity, reciprocals, FDP conversions (F 1.2 A–H).
- Recurring decimals to fractions by algebra (H 1.3 A; June 2024 2HR Q18).
- Decimals: place value, ordering, terminating decimals as fractions (F 1.3 A–E).
- Squares, cubes, roots, index laws, prime factorisation, HCF and LCM (F 1.4 A–E; June 2024 1HR Q11 family).
- Surds: simplifying, adding like surds, rationalising with conjugates (H 1.4 A–B; June 2024 1HR Q17 family).
- Sets: notation, universal/empty/complement, Venn diagrams; Higher subsets, n(A), algebraic and practical sets (F 1.5 A–E, H 1.5 A–D).
- Percentages: multipliers, percentage of, increase/decrease, reverse, compound interest and depreciation (F 1.6 A–G, H 1.6 A–B; June 2024 2HR Q8).
- Ratio in simplest form, sharing, direct proportion, word problems (F 1.7 A–E).
- Rounding to powers of ten, s.f. and d.p.; bound identification and bound problems choosing the right extremes; estimation by 1 s.f. (F 1.8 A–D, H 1.8 A; June 2024 2HR Q19).
- Standard form conversions and arithmetic (F 1.9 A, H 1.9 A; June 2024 2HR Q9).
- Everyday number, standard units and conversions, time calculations, calculator discipline (F 1.10 A–C, F 1.11 A).

Boundary notes: fractional, negative and zero indices beyond the multiplication and division laws belong to topic 2; proportion graphs belong to topics 2 and 4; compound measures (speed, density, pressure) belong to topic 4. Foundation candidates meet the Foundation rows only. Worked examples are adapted from the reviewed June 2024 papers or original; sheet questions are original unless labelled.

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Equations, formulae and identities

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4MA1 topic 2 (specification Issue 2, November 2017). Foundation rows 2.1–2.4, 2.6–2.8 are shared with Higher; proportion (2.5) is Higher-only, and Higher extends manipulation to fractional **indices** 指数, triple expansions, algebraic fractions, completing the square, proofs, linear–quadratic pairs and quadratic inequalities. On the June 2024 Higher papers this strand again carries the A/A* discriminators: an index equation, recurring-decimal proof, algebraic-fraction division, completing the square with a transformed minimum, and a quadratic inequality.

Must-know for topic 2 (Higher formulae sheet gives the **quadratic formula** 求根公式 only):

Family	Formulae
indices	$x^m \times x^n = x^{m+n}$, $x^m \div x^n = x^{m-n}$, $(x^m)^n = x^{mn}$, $x^0 = 1$, $x^{-n} = \dfrac{1}{x^n}$, $x^{1/2} = \sqrt{x}$
quadratics	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; factoring, or $a(x+b)^2 + c$ by completing the square
proportion	$y \propto x^n \iff y = kx^n$; inverse $y = k/x^n$

Symbols, indices and expressions

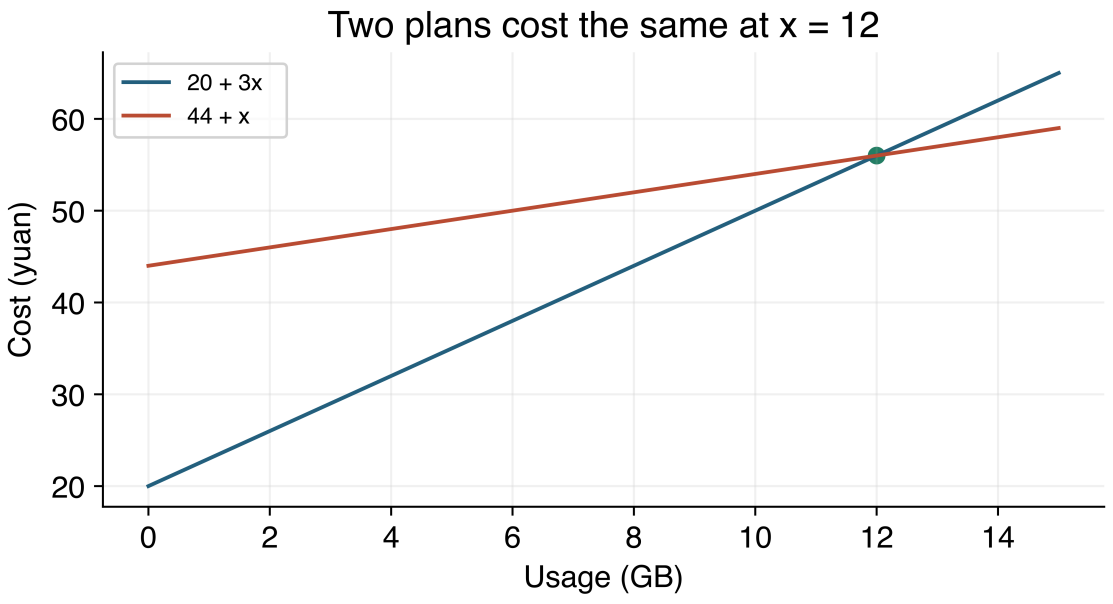
Symbols represent unknown numbers in equations or **variables** 变量 in **expressions** 表达式 and formulae; algebraic expressions follow the generalised rules of arithmetic. An **equation** 方程 asks which inputs work; an **identity** 恒等式 holds for every allowed input. **Index notation** 指数记法 covers positive, negative and zero powers: $a^3 = a \times a \times a$, $a^0 = 1$, $a^{-1} = \frac{1}{a}$, and Higher adds fractional powers: $x^{1/2} = \sqrt{x}$, $x^{1/3} = \sqrt[3]{x}$, $x^{2/3} = (\sqrt[3]{x})^2$. The **index laws** 指数法则 combine them: $x^m \times x^n = x^{m+n}$, $x^m \div x^n = x^{m-n}$, $(x^m)^n = x^{mn}$ – multiply means add the indices; divide means subtract.

> **Worked example (2HR June 2024 Q7, adapted).** $\dfrac{3^{n-2} \times 3^{n+5}}{3^{3n+10}} = 1$. Same base throughout, so the indices combine: $(n-2) + (n+5) - (3n+10) = 0$, which is $-n - 7 = 0$. Hence $n = -7$. Never multiply the 3s – the bases match, so only the indices move.

Substitution 代入 evaluates expressions (negative and fractional values included); collecting **like terms** 同类项, multiplying a single term over a bracket, taking out **common factors** 公因数 (factorise $x^2 + 10x + 24 = (x+6)(x+4)$) and **expanding** 展开 the product of two linear expressions ($(x+8)(x-5) = x^2 + 3x - 40$) are the Foundation toolkit. Higher extends expansion to three brackets: $(x+2)(x+3)(x-1) = (x^2+5x+6)(x-1) = x^3 + 4x^2 + x - 6$ – expand two first, then multiply through, and

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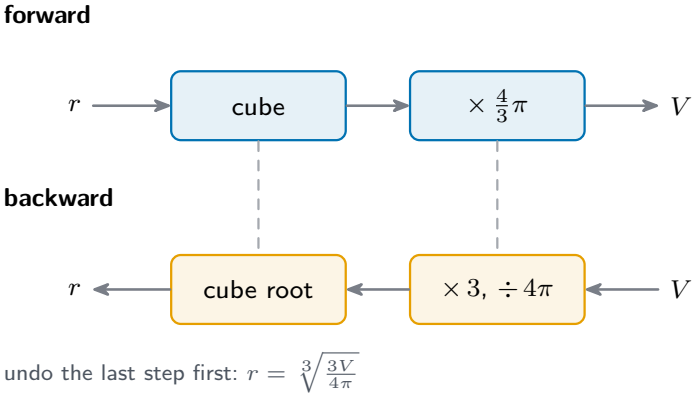
check with $x = 0$ (**constant term** 常数项 -6).



The plans cost the same where $20 + 3x = 44 + x$: $x = 12$, cost 56 yuan

Formulae and changing the subject

Use formulae from mathematics and other contexts, convert worded rules into symbols, and derive expressions from a situation. **Changing the subject** 变换主项 applies the same operation to both sides until the chosen letter stands alone: from $A = \pi r^2$, $r = \sqrt{A/\pi}$ (positive root – a length); from $v = u + at$, $t = \frac{v-u}{a}$.



Changing the subject reverses the machine: undo the last operation first, so r is the cube root of 3V over 4 pi

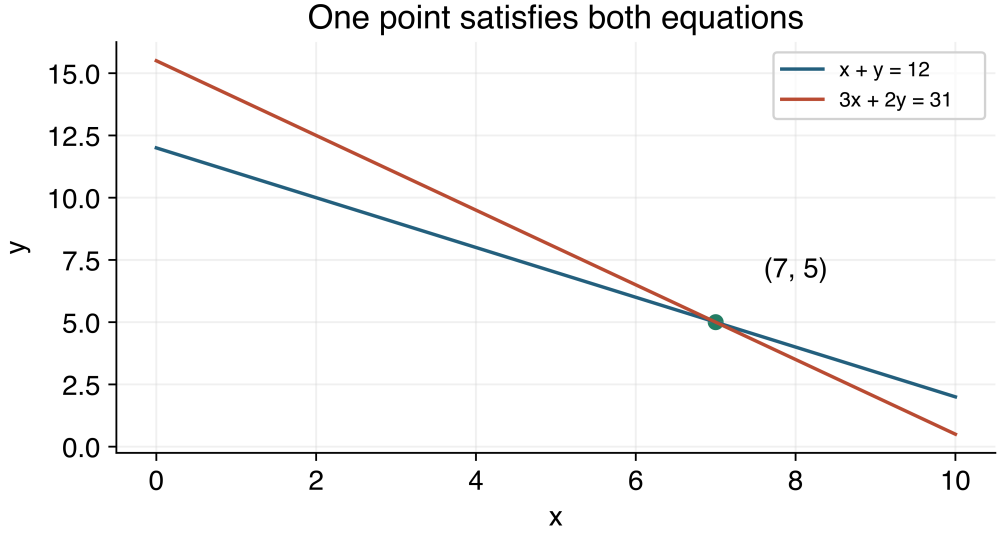
Higher cases put the subject deeper in the formula. From $V = \frac{4}{3}\pi r^3$: multiply by 3, divide by 4π, then cube-root: $r = \sqrt[3]{\frac{3V}{4\pi}}$. From $T = 2\pi\sqrt{l/g}$: square first – $T^2 = \frac{4\pi^2 l}{g}$ – so $l = \frac{gT^2}{4\pi^2}$. When the subject appears twice, collect its terms before factorising: $4a + 3 = 5(a - r)$ expands to $4a + 3 = 5a - 5r$, so $a = 5r + 3$. Check every rearrangement by substituting numbers into both versions.

Proportion 正比例 (Higher): $y \propto x^n$ means $y = kx^n$ for a constant k; **inverse proportion** 反比例 $y \propto 1/x^n$ means $y = k/x^n$. If $y \propto x^3$ and $y = 24$ when $x = 2$, then $k = 3$, so $y = 3x^3$ and $y = 81$ when $x = 3$. Graphically, $y = kx$ is a line through the **origin** 原点; $y = k/x$ a **hyperbola** 双曲线 – the algebra and the graph must tell the same story.

Linear equations, proportion and simultaneous pairs

Solve linear equations with the unknown on either or both sides, integer or fractional **coefficients** 系数 – the moves are always expand, collect, divide: $7(x + 3) = 5x + 8$ gives $7x + 21 = 5x + 8$, so $2x = -13$ and $x = -6.5$; the fractional form $\frac{4}{5}x + 3 = \frac{2}{3}$ multiplies through by 15: $12x + 45 = 10$, so $x = -\frac{35}{12}$. Set up equations from given data: the three angles $a°, (a+10)°, (a+20)°$ of a triangle sum to 180, so $3a + 30 = 180$ and $a = 50$. Check by substitution, and keep solutions sensible for the context (a count cannot be negative or fractional).

Two **simultaneous linear equations** 联立方程 solve exactly by **elimination** 消元法 or substitution: for $2x + 3y = 17$ and $3x - 5y = 35$, triple the first and double the second: $6x + 9y = 51$, $6x - 10y = 70$; subtracting gives $19y = -19$, so $y = -1$ and $x = 10$. Check in the other equation: $3(10) - 5(-1) = 35$. ✓ Graphically each equation is a line and the common solution is their **intersection point** 交点.



The solution is the crossing point: $x = 7$, $y = 5$ satisfies both equations

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Quadratic algebra

A **quadratic expression** 二次式 has the form $ax^2 + bx + c$ with $a \neq 0$. Foundation factorises $x^2 + bx + c$; Higher factorises any: $6x^2 - 5x - 6 = (3x + 2)(2x - 3)$ – check by expanding ($6x^2 - 9x + 4x - 6$ ✓). **Difference of two squares** 平方差: $4x^2 - 9 = (2x - 3)(2x + 3)$.



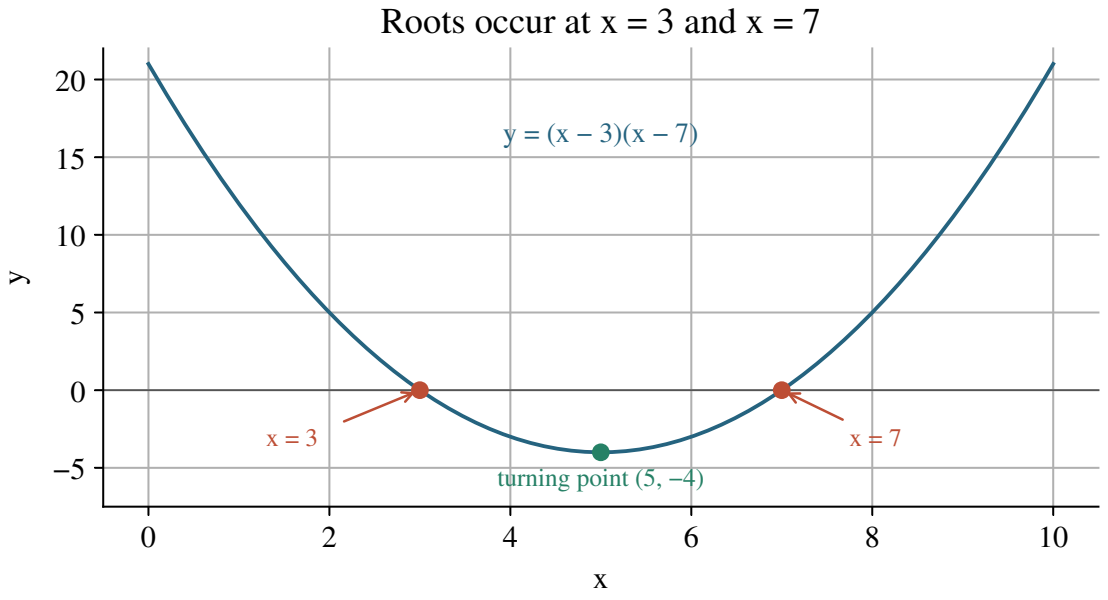
A suspension bridge cable hangs close to a parabola, the graph of a quadratic

Algebraic fractions 代数分式: factor first, then cancel whole factors, never parts of terms. Simplify $\dfrac{x^2 + 3x - 10}{x^2 + x - 2} = \dfrac{(x + 5)(x - 2)}{(x + 2)(x - 1)}$ – no cancellation here (no shared factor); but $\dfrac{x^2 - 4}{x^2 + 5x + 6} = \dfrac{(x - 2)(x + 2)}{(x + 2)(x + 3)} = \dfrac{x - 2}{x + 3}$. Adding uses a common denominator; dividing means multiplying by the **reciprocal** 倒数.

> **Worked example (2HR June 2024 Q26, adapted).** Write $4 - (3x - 5) \div \dfrac{3x^2 + x - 10}{4x - 1}$ as a single fraction. First factor the quadratic: $3x^2 + x - 10 = (3x - 5)(x + 2)$. Dividing by a fraction inverts it, and the $(3x - 5)$ cancels:
>
> $$4 - \frac{(3x - 5)(4x - 1)}{(3x - 5)(x + 2)} = 4 - \frac{4x - 1}{x + 2} = \frac{4(x + 2) - (4x - 1)}{x + 2} = \frac{9}{x + 2}$$

Proofs 证明 (Higher) use algebra, not examples. The classic **recurring decimal** 循环小数: let $x = 0.306306\ldots$; then $1000x = 306.306\ldots$, and subtracting, $999x = 306$, so $x = \frac{306}{999} = \frac{34}{111}$. State what x equals, multiply by a power of ten that shifts one full recurring block, subtract, simplify (June 2024 2HR Q18).

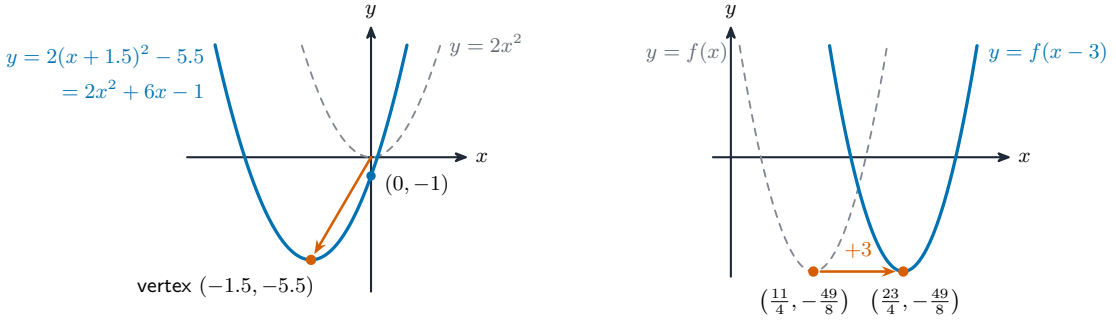
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$x^2 - 10x + 21 = (x - 3)(x - 7)$: roots $x = 3$ and $x = 7$

Completing the square and solving quadratics

Completing the square 配方 rewrites $ax^2 + bx + c = a(x + b')^2 + c'$: half the x-coefficient gives the shift, inside the bracket, with its sign flipped. The **vertex** 顶点 form exposes the minimum (for $a > 0$) immediately: the graph of $y = a(x + b')^2 + c'$ is $y = ax^2$ **translated** 平移 by $(-b', c')$.



$a(x + p)^2 + q$ has its vertex at $(-p, q)$; $f(x - 3)$ moves every point 3 to the right

Left: completing the square moves the parabola - the vertex of y = 2x squared plus 6x minus 1 at x = -1.5. Right: the June 2024 Q20 minimum shifting right by 3 under f(x-3).

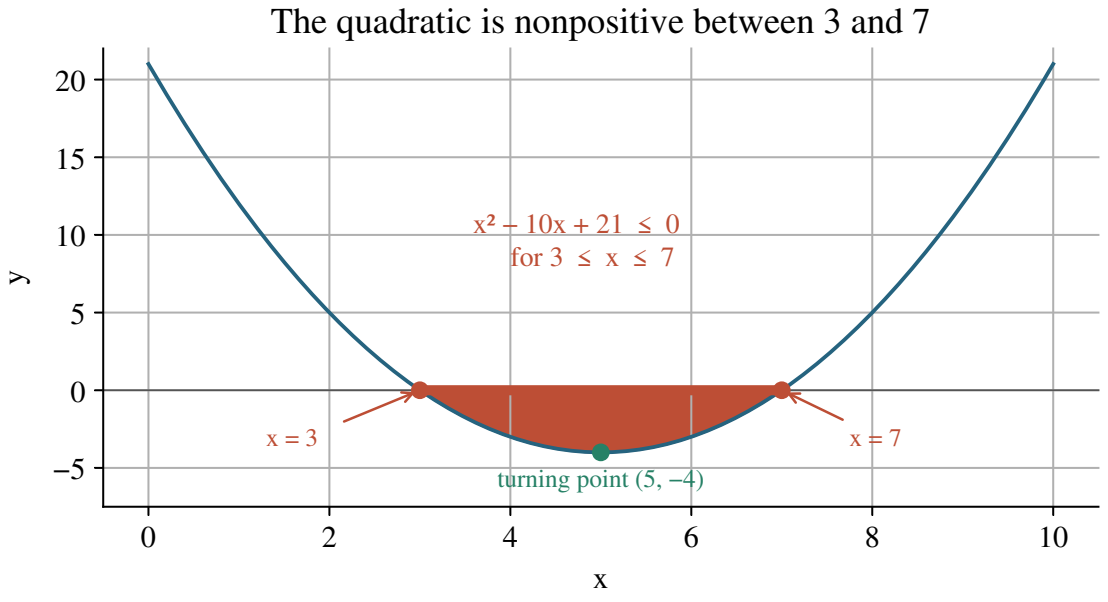
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Worked example (1HR June 2024 Q20, adapted). (a) Write $2x^2 - 11x + 9$ in the form $a(x+b)^2 + c$: factor 2 from the x terms, half of $-\frac{11}{2}$ is $-\frac{11}{4}$, so

$$2x^2 - 11x + 9 = 2\left(x - \tfrac{11}{4}\right)^2 - \tfrac{121}{8} + 9 = 2\left(x - \tfrac{11}{4}\right)^2 - \tfrac{49}{8}$$

(b) The graph of $y = f(x-3)$ shifts f three units right, so the minimum moves from $\left(\frac{11}{4}, -\frac{49}{8}\right)$ to $\left(\frac{23}{4}, -\frac{49}{8}\right) = (5.75, -6.125)$. Inside the bracket acts on x, and the other way.

Solving quadratics: by factorisation ($x^2+x-30 = (x+6)(x-5) = 0$ gives $x = -6$ or 5); by the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ when factorising fails – for $2x^2 + 6x - 1 = 0$, $x = \frac{-6 \pm \sqrt{36+8}}{4} = \frac{-3 \pm \sqrt{11}}{2}$; or by completing the square, which solves and gives the vertex at once. Form equations from context (area, Pythagoras, projectile-style $h = at^2 + bt + c$), keep exact answers unless told otherwise, and reject **roots** 根 the context forbids.



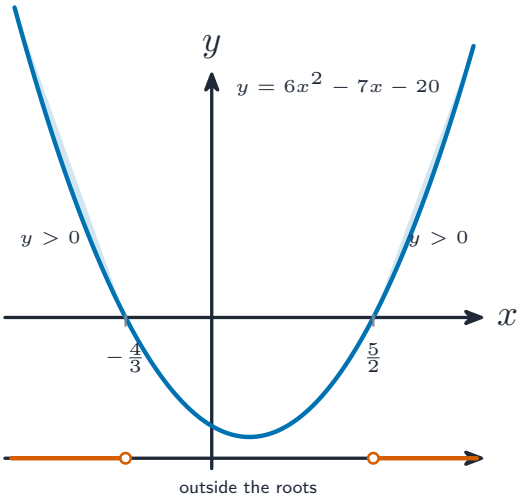
$$x^2 - 10x + 21 = (x-3)(x-7) \leq 0 \text{ for } 3 \leq x \leq 7$$

Inequalities and linear–quadratic pairs

Inequality symbols $<, >, \leq, \geq$ (double-ended forms like $1 < x \leq 5$ included) describe intervals; open and closed circles on a **number line** 数轴 mark excluded and included ends. Solving a **linear inequality** 一次不等式 mirrors equation moves – except multiplying or dividing by a negative flips the direction: $7 - x \leq 5$ gives $-x \leq -2$, so $x \geq 2$.

Quadratic inequalities 二次不等式 (Higher) need the **critical values** 临界值, then the sign of the **parabola** 抛物线 between them: a positive quadratic is above zero

outside its roots.



positive quadratic: $>$ holds outside the critical values

The June 2024 Q20 parabola y = 6x squared minus 7x minus 20 with roots -4/3 and 5/2 marked, the above-axis arcs shaded, and the outside ranges shaded on a number line below.

Worked example (2HR June 2024 Q20, adapted). Solve $6x^2 - 7x - 20 > 0$. Factorise: $(3x + 4)(2x - 5) > 0$, so the critical values are $x = -\frac{4}{3}$ and $x = \frac{5}{2}$. The parabola opens upward – positive outside the roots – so $x < -\frac{4}{3}$ or $x > \frac{5}{2}$, marked on the number line with open circles (the values themselves give 0, not > 0). Sketch the parabola first; the picture decides inside vs outside.

Linear–quadratic pairs (Higher): substitute the linear equation into the quadratic. For $y = 2x - 11$ with $x^2 + y^2 = 25$: $x^2 + (2x - 11)^2 = 25$ gives $5x^2 - 44x + 96 = 0$, and by formula $x = \frac{44 \pm \sqrt{1936 - 1920}}{10} = \frac{44 \pm 4}{10} = 4.8$ or 4, giving $(4.8, -1.4)$ and $(4, -3)$; both satisfy the circle. For $y = 11x - 2$ with $y = 5x^2$: $5x^2 - 11x + 2 = (5x - 1)(x - 2) = 0$, so $\left(\frac{1}{5}, 0.2\right)$ and $(2, 20)$. Retain both solutions unless the context removes one.

Regions (F 2.8 D–E, harder cases Higher): on a Cartesian grid, each linear inequality bounds a **half-plane** 半平面; test one point per boundary (the origin is handy unless it lies on a boundary), then shade the region satisfying every condition – for example $x \geq 0$, $y \geq 1$, $x + y \leq 5$ gives the interior of a triangle with vertices $(0, 1)$, $(0, 5)$, $(4, 1)$.

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Checklist

- Use symbols, index laws (integer, then fractional/negative/zero – Higher) and combine indices on one **base** 底数 (F 2.1, H 2.1; June 2024 2HR Q7).
- Substitute, collect terms, expand single and double brackets – and Higher, triple brackets – factorise common factors and quadratics (F 2.2, H 2.2 A–B).
- Derive, use and rearrange formulae, including subject-twice and power cases; set up and solve direct/inverse proportion (F 2.3, H 2.3, H 2.5).
- Solve linear equations from data and exact simultaneous pairs by elimination, interpreting the pair as intersecting lines (F 2.4, F/H 2.6).
- Manipulate algebraic fractions: factor, cancel, common denominators, invert to divide (H 2.2 C; June 2024 2HR Q26).
- Construct algebraic proofs, including recurring-decimal conversions (H 2.2 E; June 2024 2HR Q18).
- Complete the square, read the vertex form, and solve quadratics by factorising, formula or square – including from context (H 2.2 D, H 2.7 A–C; June 2024 1HR Q20).
- Solve linear and quadratic inequalities with number-line solution sets; identify bounded regions from linear inequalities (F 2.8, H 2.8; June 2024 2HR Q20).
- Solve linear–quadratic simultaneous pairs by substitution, keeping both points (H 2.7 D).

Boundary notes: the **discriminant** 判别式 as a named tool and factor theorem are beyond the 4MA1 list – solving uses factorising, the formula or completing the square; **surd** 根式 rationalisation appears at Higher (rationalise $\frac{7}{4(2-\sqrt{3})}$-style denominators by multiplying top and bottom by the **conjugate** 共轭, e.g. to $\frac{14+7\sqrt{3}}{4}$). Foundation meets 2.1–2.4, 2.6–2.8 only. Worked examples above are adapted from the reviewed June 2024 Higher pair (see the meta's assessment evidence); sheet questions are original unless labelled.

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Sequences, functions and graphs

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4MA1 topic 3 (specification Issue 2, November 2017). Foundation rows 3.1 and 3.3 are shared with Higher; the function-notation block 3.2 and the calculus block 3.4 are Higher-only, and Higher 3.3 extends the graphs work to cubics, reciprocals, trig curves, transformations and intersections. On the June 2024 Higher papers this strand is the heaviest of the six topics (about a third of each paper) and carries the A/A* discriminators: algebraic series sums, composite functions, perpendicular lines, line–curve intersections and stationary points.

Must-know for topic 3 (Higher formulae sheet gives the series and trapezium rules only):

Family	Formulae
arithmetic	$u_n = a + (n-1)d$, $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + u_n)$
functions	$\mathrm{fg}(x) = f(g(x))$ – do g first; $\quad f^{-1}$ undoes f on a one-to-one domain
lines	gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$; $\quad$ parallel $m_1 = m_2$; $\quad$ perpendicular $m_1 m_2 = -1$
intersections	$y_1 = y_2 \iff y_2 - y_1 = 0$ – solve, then substitute back
calculus	$\dfrac{d}{dx}\left(ax^n\right) = anx^{n-1}$; $\quad$ stationary where $\dfrac{dy}{dx} = 0$

Sequences and series

A **term-to-term rule** 递推规则 says how to get the next term (add 4: 5, 9, 13, 17, …; multiply by 2: 1, 2, 4, 8, …); a **position-to-term rule** 位值规则 gives a term directly from its position. For an **arithmetic sequence** 等差数列 with first term a and **common difference** 公差 d, the nth term is $u_n = a + (n-1)d$: for 1, 3, 5, 7, … the rule is $2n - 1$, and "nth term is $4n + 3$" generates 7, 11, 15, … Check a proposed rule on two positions before trusting it.

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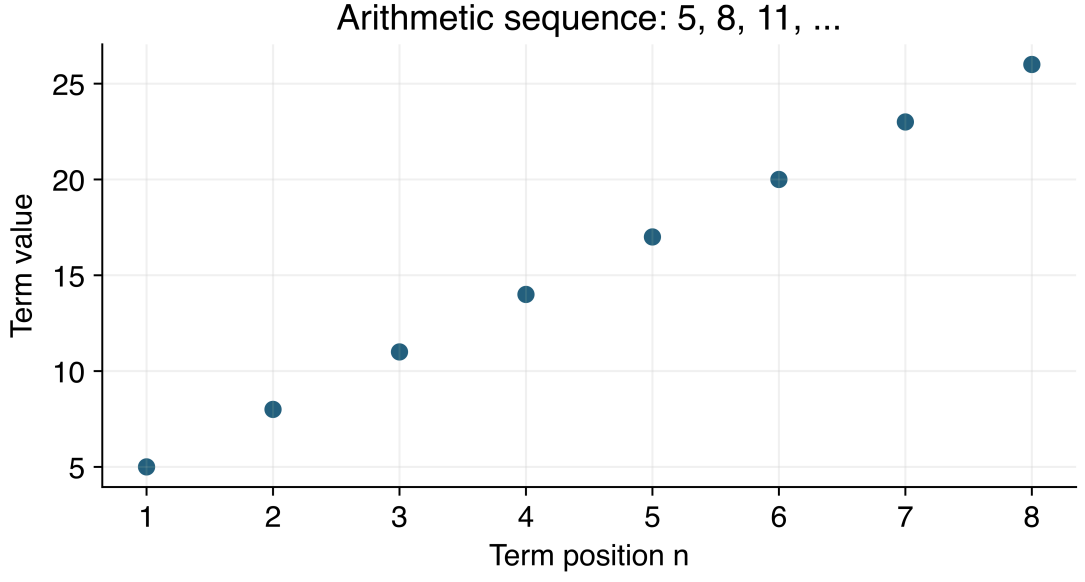


Romanesco broccoli: its spirals follow a repeating pattern, like the terms of a sequence

Higher adds the **sum of an arithmetic series** 等差级数求和. Pairing first with last, second with second-last, … gives $n/2$ equal pair-sums of $a + u_n$:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + u_n)$$

For $4 + 7 + 10 + 13 + \cdots$: $a = 4$, $d = 3$, so $S_{50} = 25\,(8 + 49 \times 3) = 25 \times 155 = 3875$. Recovering a and d from two terms uses the term rule as simultaneous equations: if $u_2 = 7$ and $u_5 = 19$, then $3d = 12$, so $d = 4$ and $a = 7 - 4 = 3$.



Add 3 each time: nth term 3n + 2, so the eighth term is 26

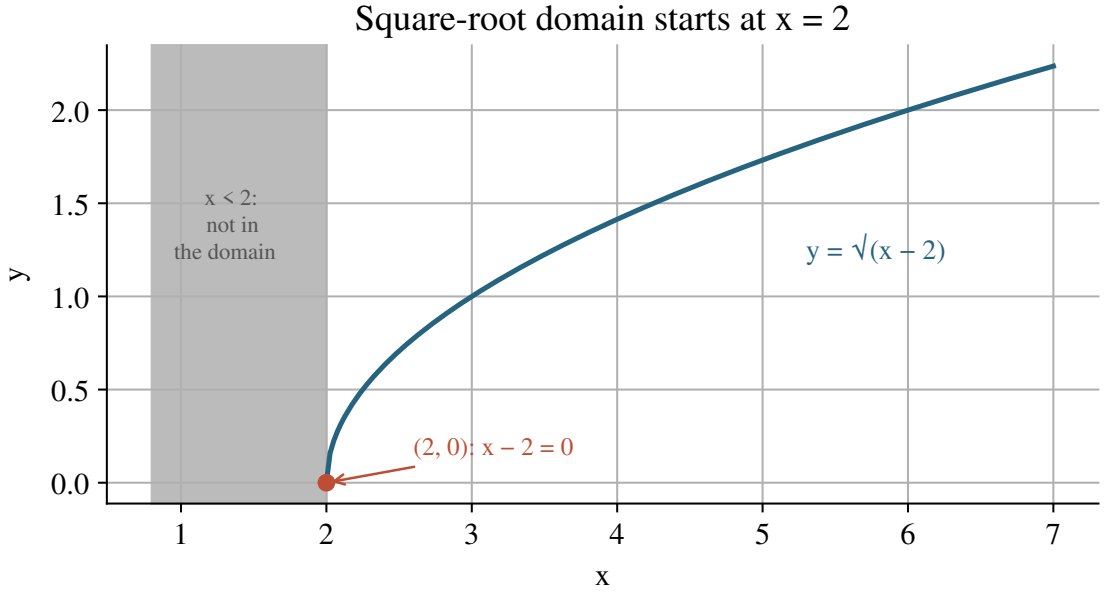
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Worked example (1HR June 2024 Q23, adapted). Three consecutive terms of an arithmetic series are $4x-14$, $x+2$ and $7x-9$. Consecutive terms carry a common difference, so $(x+2)-(4x-14) = (7x-9)-(x+2)$, giving $-3x+16 = 6x-11$, so $x = 3$. The terms are $-2, 5, 12, \ldots$: $a = -2$, $d = 7$. Then $S_{40} = 20\,[2(-2) + 39 \times 7] = 20 \times 269 = 5380$. Keep a, d symbolic until the final substitution – the algebra, not the arithmetic, carries the marks.

Function notation

A **function** 函数 maps each input of one set to exactly one output of another: write $f(x) = \ldots$ or $f\colon x \mapsto \ldots$ The **domain** 定义域 is the set of allowed inputs; the **range** 值域 is the set of outputs. Exclude inputs that break the rule: for $f(x) = \dfrac{1}{x-2}$ exclude $x = 2$; for $f(x) = \sqrt{x}$ take $x \geq 0$.

The **composite function** 复合函数 $\mathrm{fg}(x) = f(g(x))$ applies g first, then f – the output of g must be an allowed input of f. The **inverse function** 反函数 f^{-1} undoes f, and exists only where f is one-to-one: for $f(x) = \sqrt{x-2}$ (domain $x \geq 2$, range $f \geq 0$), writing $y = \sqrt{x-2}$ and squaring gives $x = y^2 + 2$, so $f^{-1}(x) = x^2 + 2$ on domain $x \geq 0$. Always check $f^{-1}(f(x)) = x$.



$y = \sqrt{(x-2)}$ is defined only for $x \geq 2$

Worked example (2HR June 2024 Q17, adapted). $f(x) = \dfrac{x}{2x-4}$ and $g(x) = 3x + 1$; find k given $\mathrm{fg}(k) = 2$. First $g(k) = 3k + 1$, then

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$$\mathrm{fg}(k) = f(3k+1) = \frac{3k+1}{2(3k+1)-4} = \frac{3k+1}{6k-2} = 2$$

so $3k+1 = 12k-4$, giving $9k = 5$ and $k = \frac{5}{9}$. (Shortcut worth knowing: solving $f(x) = 2$ first gives $x = 8/3$, so set $g(k) = 8/3$ – same answer with less algebra.)

Coordinates and straight lines

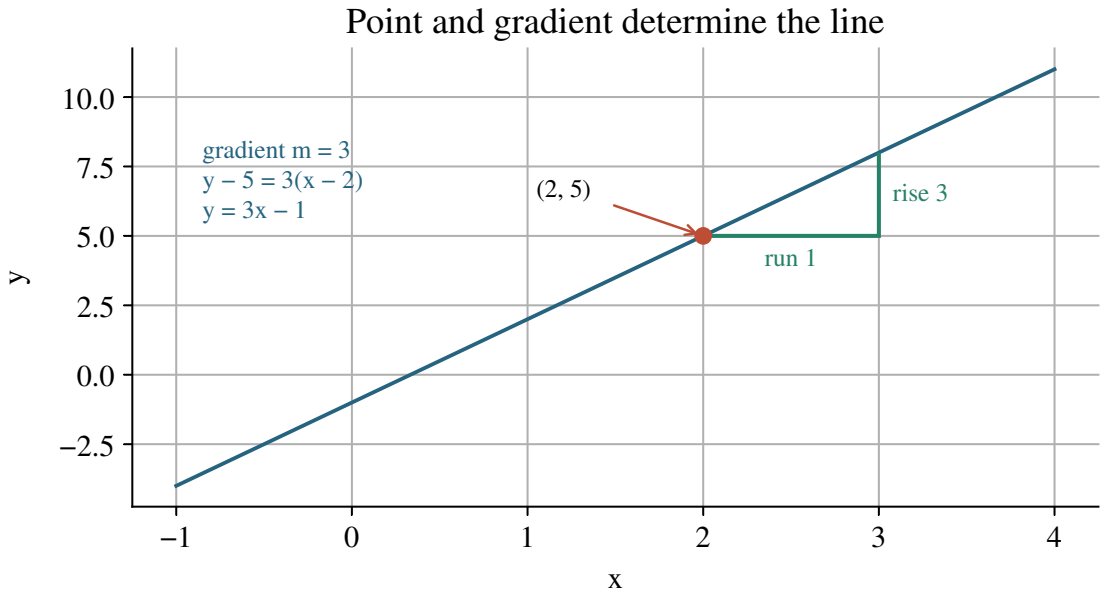
Cartesian coordinates name points in all four quadrants; a **midpoint** 中点 averages the end coordinates: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. The **gradient** 斜率 of a straight line is change in y divided by change in x: through $(1, 7)$ and $(2, 9)$ the gradient is 2. A line $y = mx + c$ has gradient m and crosses the y-axis at $(0, c)$ – for $y = 3x + 5$, gradient 3 and intercept $(0, 5)$. Special cases: $x = k$ is vertical (no finite gradient), $y = c$ is horizontal (gradient 0). Conversion graphs (currency, temperature) are read the same way.



A steep mountain road: gradient is rise over run

Interpreting real graphs is part of this block: on a distance–time graph the gradient is speed; on a speed–time graph the gradient is acceleration and the area under the graph is distance travelled. Plotting practice covers filling tables of values for straight lines (including forms $ax + by = c$) and quadratics, drawing a smooth curve through plotted points – never line segments between them.

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Gradient 3 through (2, 5): $y - 5 = 3(x - 2)$, so $y = 3x - 1$

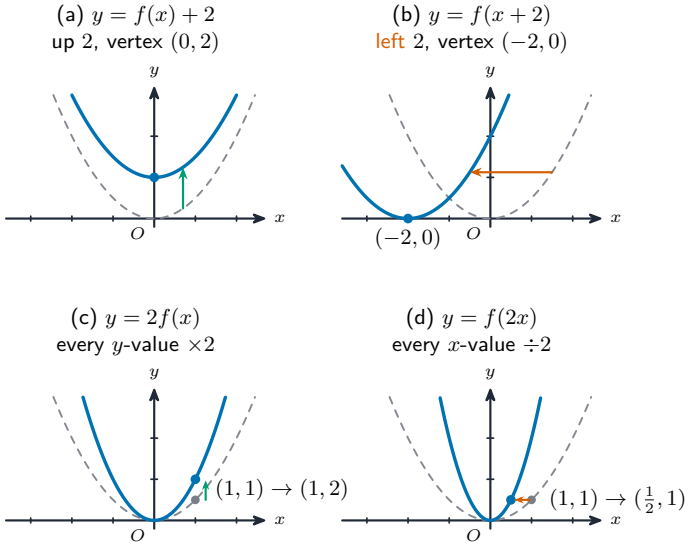
Parallel lines 平行线 share a gradient. **Perpendicular lines** 垂直线 have gradients multiplying to -1: perpendicular to m is $-1/m$. The line through (x_1, y_1) with gradient m is $y - y_1 = m(x - x_1)$.

> **Worked example (2HR June 2024 Q21, adapted).** $ABCD$ is a square with $A(-5, 2)$ and $B(3, 5)$; find the equation of BC in the form $ax + by + c = 0$. Adjacent square sides are perpendicular. Gradient of AB is $\frac{5-2}{3-(-5)} = \frac{3}{8}$, so gradient of BC is $-\frac{8}{3}$. Through $B(3, 5)$: $y - 5 = -\frac{8}{3}(x - 3)$, so $3y - 15 = -8x + 24$, giving $8x + 3y - 39 = 0$. Multiply out at the end so a, b, c are integers, as asked.

Graphs and transformations

Higher 3.3 expects recognition and plotting of three curve families beyond quadratics: cubics $y = Ax^3 + Bx^2 + Cx + D$ (any signs; $y = x^3 - 4x$ rises–falls–rises), reciprocals $y = A/x$, $y = A/x^2$, $y = A/x^3$ (two branches, asymptotes, never crossing the axes) and the trigonometric graphs $y = \sin x$, $y = \cos x$, $y = \tan x$ for angles of any size in degrees – sine and cosine oscillate between -1 and 1 with period 360°; tangent repeats every 180° and breaks at $90° + 180°n$. A circle centred at the origin has $x^2 + y^2 = r^2$ – the circle enters this course as the second curve of an intersection question.

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outside the bracket acts on y as you expect; inside acts on x the opposite way: $+2$ moves left, $\times 2$ halves

The four required transformations of a parabola - translate up, translate left, stretch vertically, compress horizontally - each drawn against the dashed original.

Four **transformations** 变换 of $y = f(x)$ are required, for linear, quadratic, sine and cosine functions:

Form	**Effect on** $y = f(x)$
$y = f(x) + a$	translate by a vertically (up if $a > 0$)
$y = f(x + a)$	translate by $-a$ horizontally (left if $a > 0$)
$y = af(x)$	vertical stretch, scale factor a
$y = f(ax)$	horizontal stretch, scale factor $1/a$

The inside forms act "backwards" on x and only on x. For $f(x) = x^2$: $y = f(x-2)+5 = (x-2)^2 + 5$ moves the minimum from $(0, 0)$ to $(2, 5)$; $y = f(2x) = 4x^2$ halves the x-values, making the parabola narrower, not taller. For $y = \sin 2x$ the period halves to 180°. Read a drawn transformation off the graph (where did the turning point go?) and write it algebraically – both directions are assessed.

Gradients of curves: draw a tangent at the point and take its gradient as rise/run – the drawn line, not the curve, is measured. Exact curve gradients need calculus (next section).

Intersections: the x-values where a linear y_1 meets a non-linear y_2 are the solutions of $y_2 - y_1 = 0$. Meeting points of $y = 2x + 1$ and $y = x^2 + 3x - 2$ solve $x^2 + x - 3 = 0$; of $y = 5$ and $y = x^3 - 3x^2 + 7$, solve $x^3 - 3x^2 + 2 = 0$.

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Worked example (2HR June 2024 Q22, adapted). Find where the line $y = 3x - 1$ meets the curve $x^2 + y^2 = y + 11$. Substitute the line into the curve:

$$x^2 + (3x-1)^2 = (3x-1) + 11 \;\Rightarrow\; 10x^2 - 9x - 9 = 0 \;\Rightarrow\; (5x+3)(2x-3) = 0$$

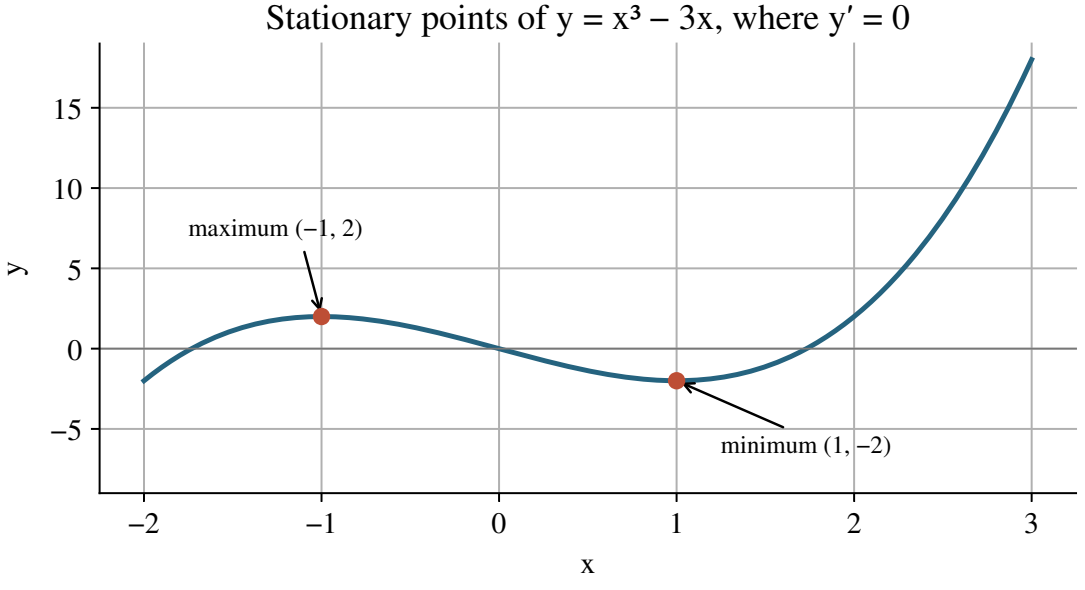
so $x = -\frac{3}{5}$ or $x = \frac{3}{2}$, with $y = 3x - 1$ giving $y = -\frac{14}{5}$ or $y = \frac{7}{2}$. The points are $\left(-\frac{3}{5}, -\frac{14}{5}\right)$ and $\left(\frac{3}{2}, \frac{7}{2}\right)$ – substitute back into the *other* equation to check: $\frac{9}{4} + \frac{49}{4} = \frac{58}{4} = 14.5 = \frac{7}{2} + 11$. ✓

Differentiation and stationary points

The **gradient of a curve** varies along it: a **variable rate of change** 变化率. **Differentiation** 求导 gives it exactly: for integer powers of x,

$$y = ax^n \;\Rightarrow\; \frac{dy}{dx} = anx^{n-1}, \qquad \text{term by term}$$

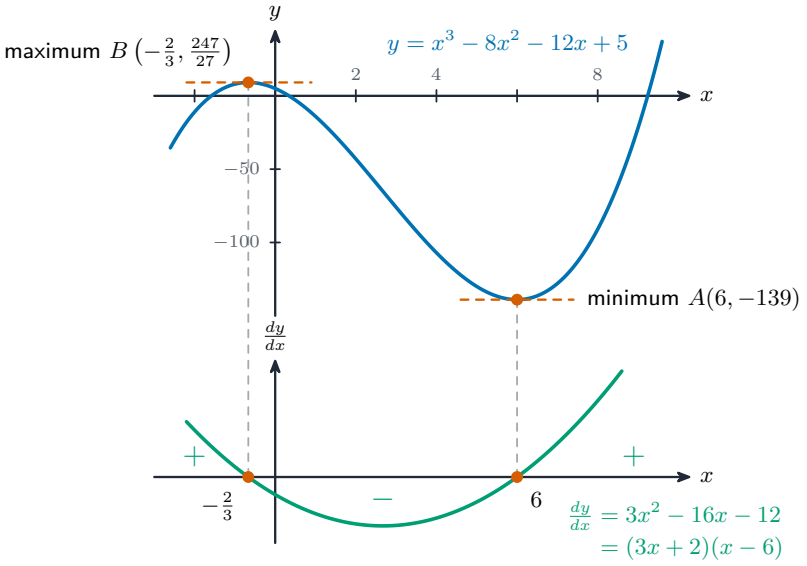
For $y = x^3 - 8x^2 - 12x + 5$: $\dfrac{dy}{dx} = 3x^2 - 16x - 12$. A **stationary point** 驻点 has $\dfrac{dy}{dx} = 0$: here $3x^2 - 16x - 12 = (3x+2)(x-6)$, so $x = -\frac{2}{3}$ or $x = 6$. Distinguish **maximum** 极大 from **minimum** 极小 from the graph's shape: a cubic with positive x^3 coefficient rises–falls–rises... here $y \to -\infty$ as $x \to -\infty$, so the left stationary point $x = -\frac{2}{3}$ is the maximum ($y = \frac{247}{27} \approx 9.1$) and the right one is the minimum ($y(6) = 216 - 288 - 72 + 5 = -139$).



$y' = 3x^2 - 3 = 0$ at $x = \pm 1$: maximum $(-1, 2)$, minimum $(1, -2)$

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Worked example (1HR June 2024 Q19, adapted). C has equation $y = x^3 - 8x^2 - 12x + 5$ and exactly two stationary points A and B with $x_A > x_B$; find A. Differentiate and set to zero: $(3x + 2)(x - 6) = 0$, and the larger root is $x = 6$. Substituting into the curve: $y = 216 - 288 - 72 + 5 = -139$, so $A = (6, -139)$. Marks follow the chain – differentiate (at least two terms right), set to zero, solve the quadratic, substitute back.



stationary where $\frac{dy}{dx} = 0$: + then − is a maximum, − then + a minimum

Top: the June 2024 Q19 cubic y = x cubed minus 8 x squared minus 12 x plus 5, with the maximum B(-2/3, 247/27) and the minimum A(6, -139) marked by horizontal tangents. Below it, on the same x-scale, the gradient dy/dx = (3x + 2)(x - 6) is zero at x = -2/3 and x = 6, positive, then negative, then positive.

Kinematics: with displacement s in metres after t seconds, velocity is the rate of change of displacement and acceleration the rate of change of velocity: $v = \dfrac{ds}{dt}$, $a = \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2}$. For $s = t^3 - 24t$ (metres, $0 \le t \le 20$): $v = 3t^2 - 24$ m/s and $a = 6t$ m/s^2. The particle is instantaneously at rest where $v = 0$: $t = \sqrt{8} = 2.83$ s (3 s.f.).

Checklist

- Generate sequences from term-to-term and position-to-term rules; find the rule of an integer sequence (F 3.1 A–B).
- Use $u_n = a + (n - 1)d$ for arithmetic sequences; find a and d from any two terms (F 3.1 C, H 3.1 A–B).

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- Sum an arithmetic series with $S_n = \frac{n}{2}[2a + (n-1)d]$, including algebraic terms (H 3.1 C; June 2024 1HR Q23).
- Use $f(x)$ and $f: x \mapsto$ notation; state domains with exclusions and ranges (H 3.2 A–C).
- Build $\mathrm{fg}(x)$ – g first – and find f^{-1} on a one-to-one domain (H 3.2 D; June 2024 2HR Q17).
- Read, plot and interpret graphs in all quadrants: midpoints, conversion graphs, distance/speed–time graphs, tables of values (F 3.3 A–I).
- Gradients from two points; $y = mx + c$; equations of parallel and perpendicular lines through a point (F 3.3 G–H, H 3.3 F–G; June 2024 2HR Q21).
- Recognise and sketch cubics, reciprocal curves, sin / cos / tan, and the origin-centred circle $x^2 + y^2 = r^2$ (H 3.3 A).
- Apply and read off the four transformations $f(x) + a$, $f(x + a)$, $af(x)$, $f(ax)$ (H 3.3 B–C).
- Estimate a curve's gradient by drawing a tangent (H 3.3 D).
- Solve linear–non-linear intersections via $y_2 - y_1 = 0$ and check by substitution (H 3.3 E; June 2024 2HR Q22).
- Differentiate integer powers; find stationary points; classify max/min from the graph shape (H 3.4 A–D; June 2024 1HR Q19).
- Apply calculus to motion: $v = ds/dt$, $a = dv/dt$, and "at rest" means $v = 0$ (H 3.4 E).

Boundary notes: geometric series and sum-to-infinity are excluded at this level (no geometric rows in 4MA1 3.1); differentiation is limited to integer powers of x – chain, product and quotient rules are excluded; radian measure is excluded (degrees throughout). Foundation candidates meet 3.1 and 3.3 only. Worked examples above are adapted from the reviewed June 2024 Higher pair (see the meta's assessment evidence); all other questions on the sheets are original.

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Geometry and trigonometry

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4MA1 topic 4 (specification Issue 2, November 2017) is the largest topic: Foundation rows 4.1–4.11 cover angle facts, polygons, measures, constructions, circle language, Pythagoras and 2D trigonometry, mensuration and similarity; Higher adds the intersecting chord properties, the full circle-theorem reasoning programme, obtuse-angle and non-right-angled trigonometry, 3D problems, sectors, cones, spheres and the k^2/k^3 similarity ratios. On the June 2024 Higher papers this strand supplied the multi-step AO3 chains: a reasoned circle-theorem chase, a composite hexagon area, a 3D cosine-rule angle and a k^3 similar-solids equation.

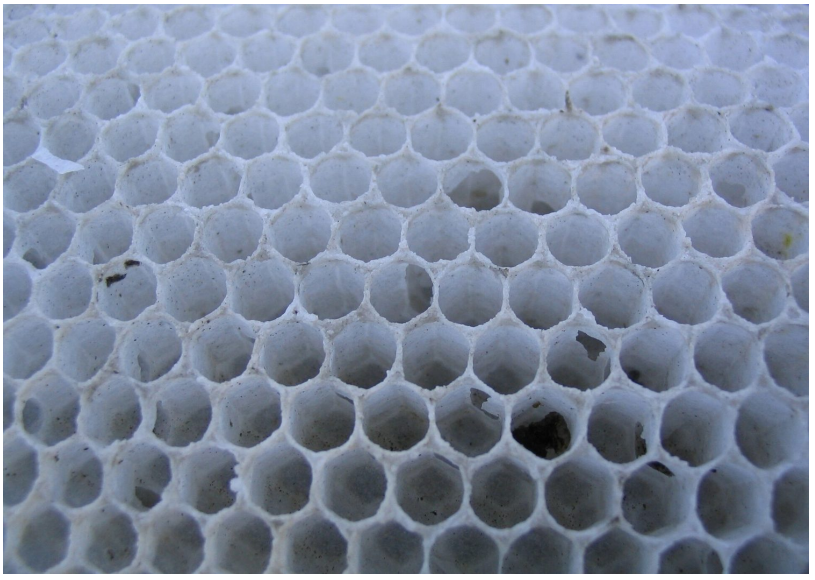
Must-know for topic 4 (Higher formulae sheet gives the sine/cosine rules, ½ab sin C, and the cone/sphere volume and surface formulae):

Family	Formulae
triangles	$a^2 = b^2 + c^2 - 2bc\cos A$; $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$; area $= \frac{1}{2}ab\sin C$
circle	arc $= \frac{\theta}{360} \times 2\pi r$; sector area $= \frac{\theta}{360} \times \pi r^2$ (θ in degrees)
solids	cylinder $V = \pi r^2 h$; cone $V = \frac{1}{3}\pi r^2 h$; sphere $V = \frac{4}{3}\pi r^3$, $SA = 4\pi r^2$
similarity	lengths k : areas k^2 : volumes k^3

Angles, polygons and symmetry

Angles classify as acute ($< 90°$), right, obtuse (90°–180°) and reflex ($> 180°$). Angles on a straight line sum to 180°, around a point to 360°, vertically opposite angles are equal, and parallel lines give equal **corresponding angles** 同位角 and **alternate angles** 内错角 (allied angles sum to 180°). The exterior angle of a triangle equals the sum of the two remote interior angles, and the angle sum is 180°; an isosceles triangle has two equal sides and base angles, an equilateral three of each at 60°.

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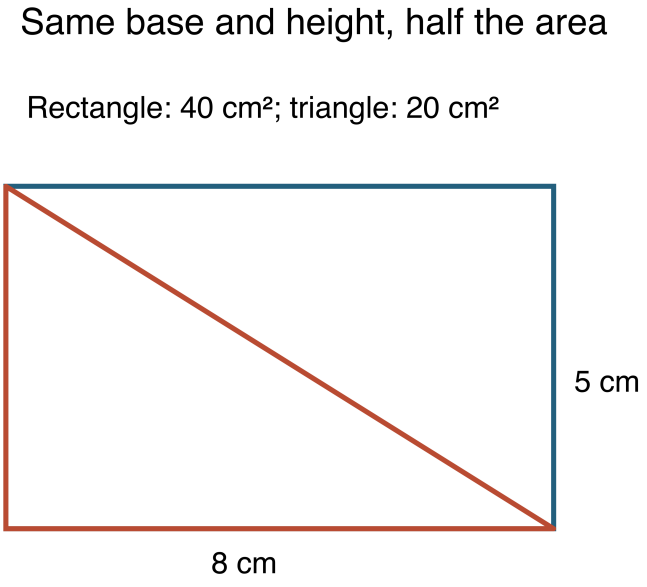


A honeycomb: regular hexagons tessellate because three interior angles of 120° meet at each point

Polygons carry names by side count; a **quadrilateral** 四边形 has an angle sum of 360°, and each polygon's interior angles sum to $(n-2)\times 180°$. A **regular polygon** 正多边形 has all sides and angles equal: exterior angle $= 360°/n$, interior $= 180° - 360°/n$ – a regular pentagon has interior angles of 108°. The special quadrilaterals nest: parallelogram (opposite sides and angles equal, diagonals bisect), rectangle (also right angles), rhombus (also all sides equal, diagonals perpendicular), square, kite, trapezium (one parallel pair). Two figures are **congruent** 全等 when identical in shape and size (conditions SSS, SAS, ASA, RHS); **similar** 相似 figures have equal angles with corresponding sides in ratio.

A shape's **line of symmetry** 对称轴 reflects it onto itself; its **order of rotational symmetry** 旋转对称 is the number of self-matches in one turn (a rectangle: 2 lines, order 2; a regular hexagon: 6 lines, order 6).

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Rectangle $8 \times 5 = 40$ cm^2; the triangle with the same base and height is half: 20 cm^2

Measures, bearings and compound rates

Read instrument scales sensibly and estimate before computing; time intervals work in both 12- and 24-hour clocks. An angle measure includes **three-figure bearings** 方位角: measured clockwise from north, written as three digits – 040°, not 40°. The return bearing reverses: if Q bears 040° from P, then P bears 220° from Q (add 180°, taking the answer mod 360).

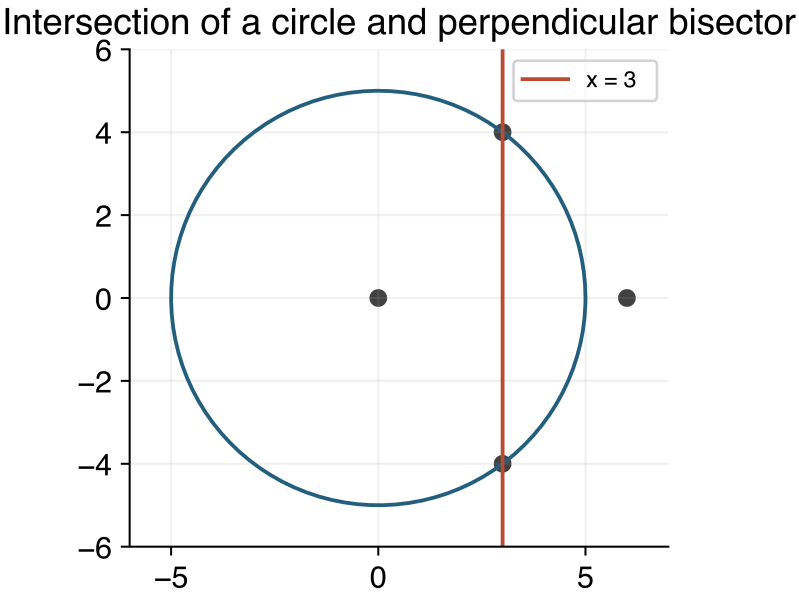
Average **speed** 速度 is distance over time; **density** 密度 is mass over volume; **pressure** 压强 is force over area – all one family of **compound measures** 复合量: fix the unit vocabulary (km/h, g/cm^3, N/m^2) and convert carefully. A car covering 234 km in 2 h 36 min averages $234 \div 2.6 = 90$ km/h.

Constructions and scale drawings

Measure and draw lines to the nearest millimetre; construct triangles from SSS, SAS or ASA with ruler and protractor. With **straight edge and compasses** construct (i) the perpendicular bisector of a line segment and (ii) the bisector of an angle; these also build specific angles (60° from an equilateral arc construction, 90° by perpendicular, 45° by bisecting). Constructions answer **locus** 轨迹 questions: the set of points a fixed distance from a point (circle), from a line (parallel pair), equidistant from two points (perpendicular bisector) or from two lines (angle bisector) – shade the region satisfying several conditions at once.

Scale drawings and maps carry a ratio (1 : 50 000): convert map distance to real distance through the scale factor, keeping units explicit.

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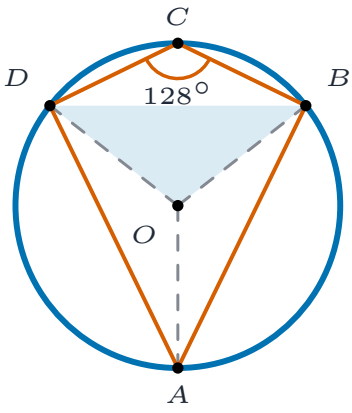


Points 5 from O and equidistant from (0, 0) and (6, 0): $x = 3$ meets the circle at (3, ± 4)

Circle geometry with reasons

Circle language: centre, radius, diameter, **chord** 弦, circumference, **tangent** 切线 (touches at one point, perpendicular to the radius there), arc, sector, segment. Foundation uses chord and tangent properties informally; Higher requires the full programme with standard reasons. The **cyclic quadrilateral** 圆内接四边形 has opposite angles summing to 180°; the angle at the centre is twice the angle at the circumference on the same arc; angles in the same segment are equal; the angle in a semicircle (standing on a diameter) is a right angle; and the angle between a tangent and a chord equals the angle in the alternate segment (the alternate segment theorem). Chord properties: the perpendicular from the centre bisects the chord; two intersecting chords satisfy $AE \times EB = CE \times ED$, and the internal/external versions power this family.

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cyclic quad: $\angle BAD = 52^\circ$

centre $= 2\times$ rim: $\angle BOD = 104^\circ$

$OB = OD$ (radii): isosceles

$\angle OBD = \frac{180-104}{2} = 38^\circ$

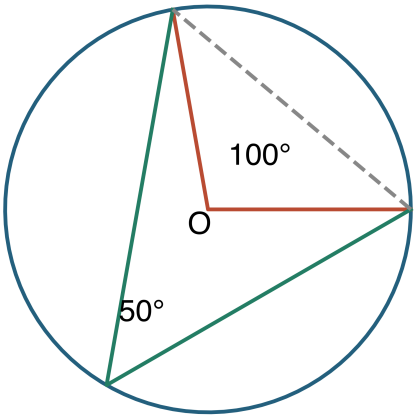
every numerical step names its reason

The June 2024 Q13 configuration: a circle centre O with cyclic quadrilateral ABCD, angle BCD marked 128 degrees, and the chain BAD = 52, BOD = 104, OBD = 38 written beside it.

Worked example: 1HR June 2024 Q13, adapted. A, B, C, D lie on a circle centre O with $\angle BCD = 128°$; find $\angle OBD$. Opposite angles of the cyclic quadrilateral sum to 180°, so $\angle BAD = 180 - 128 = 52°$. The angle at the centre is twice the angle at the circumference on the same arc: $\angle BOD = 104°$. Triangle OBD is isosceles ($OB = OD$, radii), so $\angle OBD = \frac{180-104}{2} = 38°$. Every numerical step names its reason – that is what the marks list pays for.

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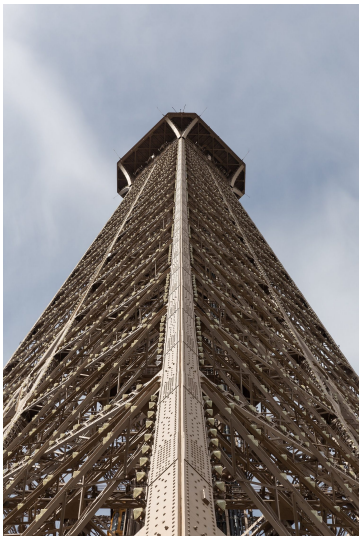
The centre angle is twice the circumference angle



The angle at the centre is twice the angle at the circumference: 100° and 50°

Pythagoras and trigonometry in 2D

Pythagoras' theorem 勾股定理: $a^2 + b^2 = c^2$ in right-angled triangles (6-8-10 is a whole-number triple; 5-12-13 another). SOH CAH TOA gives $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$ – used forwards for sides and backwards (Shift-sin) for angles. The **angle of elevation** 仰角 rises from the horizontal, the **angle of depression** 俯角 falls from it (alternate angles carry it into the triangle).



Looking up at a tower: the angle of elevation and the distance give the height using tan

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Higher extends the ratios to **obtuse angles**: $\sin 130^\circ = \sin 50^\circ$ (sine stays positive), $\cos 130^\circ = -\cos 50^\circ$ (cosine turns negative). Any triangle now yields to the **sine rule** 正弦定理 $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$ (a pair – side over opposite angle – plus one more part) or the **cosine rule** 余弦定理 $a^2 = b^2 + c^2 - 2bc\cos A$ (two sides and the included angle, or three sides), and the area formula $\frac{1}{2}ab\sin C$ works in reverse: area 15 with $a = 6$, $b = 10$ gives $\sin C = \frac{2\times 15}{60} = 0.5$, so $C = 30^\circ$ (or 150° if the diagram shows an obtuse angle).

Worked example: sine rule. In triangle ABC, $\angle A = 40^\circ$, $\angle B = 65^\circ$ and $a = BC = 8$ cm. Find $b = AC$. Use the pair side-over-opposite-angle twice:

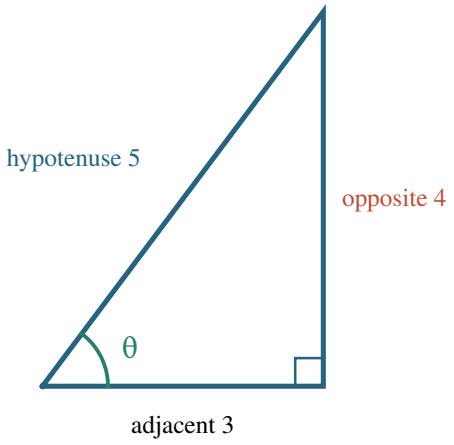
$$\frac{b}{\sin 65^\circ} = \frac{8}{\sin 40^\circ} \;\Rightarrow\; b = \frac{8\sin 65^\circ}{\sin 40^\circ} = 11.3 \text{ cm}.$$

Worked example: cosine rule with an obtuse angle. Two sides 7 cm and 9 cm meet at 110°. Find the third side a:

$$a^2 = 7^2 + 9^2 - 2(7)(9)\cos 110^\circ = 130 + 43.1 = 173.1, \qquad a = 13.2 \text{ cm}.$$

Because $\cos 110^\circ$ is negative, the last term is **added**; the side opposite an obtuse angle is the longest. Check the calculator is in degree mode.

Label the known sides before choosing a rule



opposite: across from θ
adjacent: next to θ
hypotenuse: across from the right angle

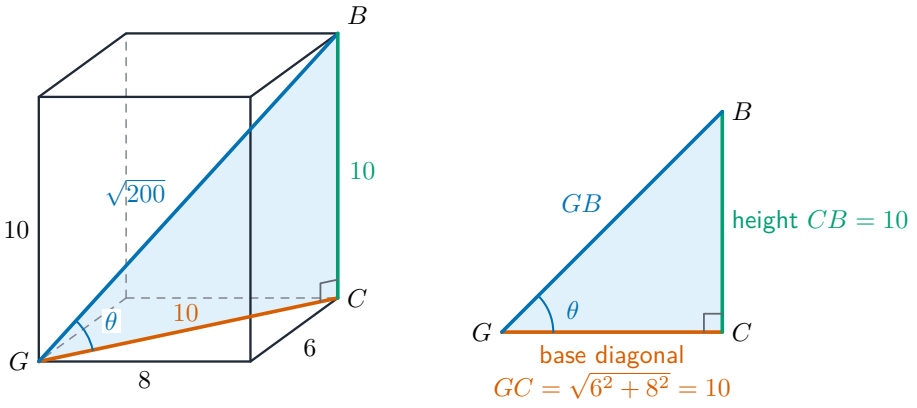
tan θ = 4/3 sin θ = 4/5
cos θ = 3/5

Label opposite, adjacent and hypotenuse first: 3, 4 and 5 here

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Trigonometry in three dimensions

3D problems are 2D problems inside a solid: name the right triangle, then Pythagoras or trigonometry as usual. In a cuboid with base 6 cm × 8 cm and height 10 cm, the base diagonal is $\sqrt{6^2+8^2}=10$ cm, and the **space diagonal** 长方体对角线 is $\sqrt{6^2+8^2+10^2}=\sqrt{200}\approx 14.1$ cm; the angle between the space diagonal and the base diagonal satisfies $\tan\theta=\frac{10}{10}$, so $\theta=45°$.



find the right triangle GCB: $GB=\sqrt{200}\approx 14.1$, $\tan\theta=1$, $\theta=45°$

A cuboid 6 by 8 by 10 with the base diagonal 10, the space diagonal root 200 and the 45 degree angle between them marked; the right triangle GCB is drawn again flat beside it.

Worked example: 1HR June 2024 Q22, adapted. Points B, M, P in a solid box have $BM=\sqrt{200}$, $BP=\sqrt{244}$ and $MP=10$ (each length itself found by Pythagoras in a right triangle of the box). Angle BMP needs the cosine rule on triangle BMP:

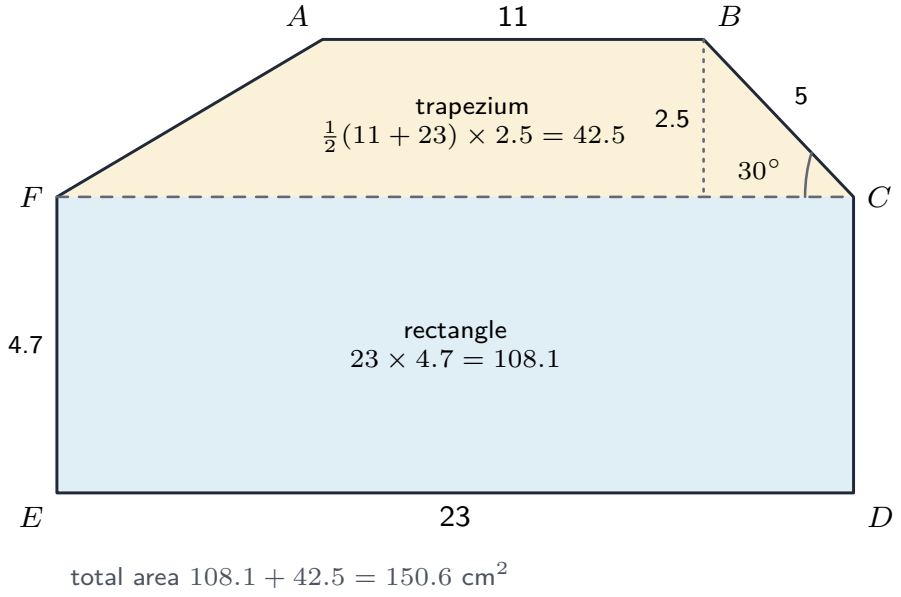
$$\cos BMP=\frac{200+100-244}{2\times\sqrt{200}\times 10}=\frac{56}{282.8}=0.198,\qquad \angle BMP=78.6°$$

Order of attack: identify the target triangle, find its three sides (each possibly by its own Pythagoras), then apply the cosine or sine rule, keeping the angle you want at the centre of the notation.

Mensuration: perimeter, area, surface and volume

Metric conversions: linear cm↔m by 100, area by 100^2, **volume** 体积 by 100^3 ($1\text{ m}^3=10^6\text{ cm}^3$). Perimeters and areas build from triangles ($\frac{1}{2}bh$), rectangles, parallelograms (bh) and trapezia ($\frac{1}{2}(a+b)h$); the circle has circumference $2\pi r$ and area πr^2 (semicircles included). Composite shapes split into these parts.

Worked example: 2HR June 2024 Q10, adapted. Hexagon $ABCDEF$ has $AB \parallel FC \parallel ED$ with $AB = 11$, $ED = FC = 23$, the perpendicular distance between FC and ED equal to 4.7 (cm) and the slant side $BC = 5$ making 30° with FC. The strip between FC and ED is a rectangle; above it the height of the trapezium $ABCF$ is $5 \sin 30° = 2.5$. Split: rectangle $23 \times 4.7 = 108.1$ plus trapezium $\frac{1}{2}(11 + 23) \times 2.5 = 42.5$, so the area is 150.6 cm^2 (exact $\frac{753}{5}$). Draw the split lines before computing.



Split first: rectangle 108.1 plus trapezium 42.5 gives 150.6 square centimetres. The trapezium height is 5 sin 30 degrees = 2.5

Higher adds **sectors** 扇形: arc $= \frac{\theta}{360} \times 2\pi r$, sector area $= \frac{\theta}{360} \times \pi r^2$ – for $r = 9$, $\theta = 40°$: arc $= 2\pi \approx 6.28$, area $= 9\pi \approx 28.3$. Sector perimeter adds the two radii. Solids: prisms $V =$ cross-section $\times$ length (cuboid, cylinder $V = \pi r^2 h$, surface $2\pi r^2 + 2\pi r h$); the **cone** 圆锥 $V = \frac{1}{3}\pi r^2 h$ with curved surface $\pi r l$ (l the slant); the **sphere** 球 $V = \frac{4}{3}\pi r^3$, surface $4\pi r^2$. A cone $r = 3$, $h = 4$ has $l = 5$ (Pythagoras), $V = 12\pi$, curved surface 15π.

Similarity: length, area and volume ratios

Similar figures have corresponding lengths in a fixed ratio k (the **scale factor** 相似比): then areas scale by k^2 and volumes by k^3 – and conversely, an area ratio 4 : 9 means a length ratio 2 : 3. Maps and models use k directly.

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Nesting dolls are similar shapes: double the length and the volume becomes eight times as large

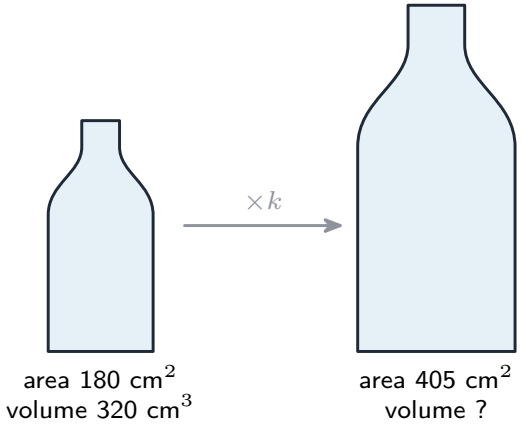
Worked example: 2HR June 2024 Q24 family, adapted. Solid B is similar to solid A with every length k times A's. Volumes then scale by k^3: the paper's cone-plus-hemisphere pair gives $k^3 = \frac{125}{8}$, so $k = \sqrt[3]{\frac{125}{8}} = \frac{5}{2} = 2.5$. A second pair with length ratio 4 has volume ratio $4^3 = 64$ – the larger solid holds 64 times the smaller. Set the ratio up on lengths first, then square or cube it once.

Worked example: area to volume. Two similar bottles have surface areas 180 cm^2 and 405 cm^2. The smaller holds 320 cm^3. Find the volume of the larger. Go through the length scale factor:

$$k^2 = \frac{405}{180} = 2.25 \;\Rightarrow\; k = 1.5 \;\Rightarrow\; k^3 = 3.375, \qquad V = 320 \times 3.375 = 1080 \text{ cm}^3.$$

The common error is to multiply the volume by the area ratio 2.25 (giving 720 cm^3).

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area: $k^2 = 405 \div 180 = 2.25$

length: $k = \sqrt{2.25} = 1.5$

volume: $k^3 = 1.5^3 = 3.375$

$320 \times 3.375 = 1080$ cm^3

Area ratio, then length ratio, then volume ratio: 2.25, 1.5, 3.375. The larger bottle holds 1080 cubic centimetres, not 720

Checklist

- Classify angles; use line, point, parallel-line and triangle angle facts, including exterior-angle and polygon sums with regular-polygon interior/exterior angles (F 4.1, 4.2).
- Identify symmetry: lines and rotational order (F 4.3).
- Use bearings, speed/distance/time, density and pressure as compound measures (F 4.4).
- Construct triangles, bisectors and perpendiculars; answer locus and scale-drawing questions (F 4.5, F 4.11 B).
- Use circle language and chord/tangent properties; Higher: cyclic quadrilaterals, centre/circumference angles, intersecting chords – every step with its standard reason (F 4.6–4.7, H 4.6–4.7; June 2024 1HR Q13).
- Pythagoras and SOH CAH TOA in 2D with elevations and depressions; obtuse ratios, sine rule, cosine rule and ½ab sin C including reversals (F 4.8, H 4.8 A–C, E).
- Solve 3D problems by extracting right triangles; cosine rule on the extracted triangle (H 4.8 D, F; June 2024 1HR Q22).
- Perimeters and areas including trapezia, circles and sectors; surface areas and volumes of prisms, cylinders, cones and spheres with unit conversions (F 4.9–4.10, H 4.9, H 4.10; June 2024 2HR Q10).
- Length : area : volume ratios $k : k^2 : k^3$, used both directions (F 4.11, H 4.11; June 2024 2HR Q24 family).

Boundary notes: bearings are three-figure; constructions are judged on visible arcs (leave them in); radians are excluded – degrees only; the ambiguous sine-rule case

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and 3D circle work are beyond the list. Foundation meets 4.1–4.11 Foundation rows only. Worked examples are adapted from the reviewed June 2024 Higher pair (see the meta's assessment evidence); sheet questions are original unless labelled.

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Vectors and transformation geometry

Pearson Edexcel · International GCSE · Mathematics A

4MA1 topic 5 (specification Issue 2, November 2017). Foundation 5.2 is the four transformations with full descriptions; 5.1 (vectors) is Higher-only and runs from notation to resultants, magnitudes and simple geometrical proofs. On the June 2024 Higher paper the vector question (2HR Q25) is the classic A/A* finisher: a chain of points expressed in **a** and **b**, then an unknown ratio k found by writing the same vector two independent ways.

Must-know for topic 5:

Family	**Formulae**
column vector	$\begin{pmatrix} x \\ y \end{pmatrix}$; scalar multiple $k\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$
magnitude 大小	$\left\lvert \begin{pmatrix} x \\ y \end{pmatrix} \right\rvert = \sqrt{x^2 + y^2}$
route rule	$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$; reversing: $\overrightarrow{BA} = -\overrightarrow{AB}$
transformations	reflection in $x = k$, $y = k$, $y = x$; rotation (centre, angle, direction); translation $\begin{pmatrix} a \\ b \end{pmatrix}$; enlargement (centre, factor k)

Vectors: notation to resultants

A **vector** 向量 has both magnitude and direction (a **scalar** 标量 has only size). Write it bold **a**, underlined, or as $\overrightarrow{OA}$; as a **column vector** 列向量 $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ – across, then up.

Scalar multiples 数乘 stretch: $2\mathbf{a}$ is twice as long and **parallel** 平行; $-\mathbf{a}$ reverses direction; $\frac{1}{2}\mathbf{a}$ halves it.

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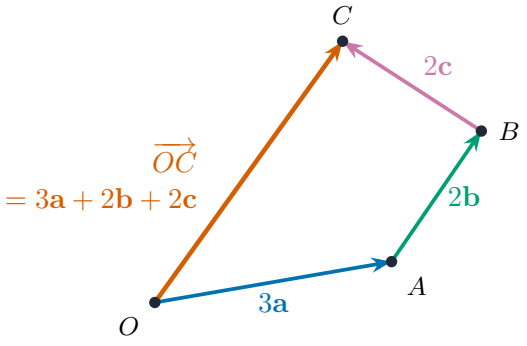
Wind and current both push a boat: vectors with size and direction add to a resultant

Addition is tip-to-tail: $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$ – the route matters, not the stops. Reverse a vector by negating: $\overrightarrow{BA}=-\overrightarrow{AB}$. The **resultant** 合向量 of several legs is the single start-to-end vector: with $\overrightarrow{OA}=3\mathbf{a}$, $\overrightarrow{AB}=2\mathbf{b}$ and $\overrightarrow{BC}=2\mathbf{c}$ (the specification's own example),

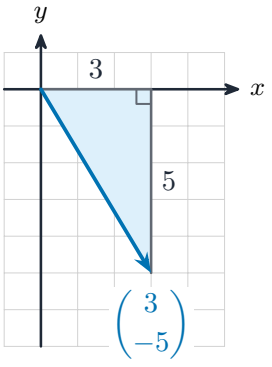
$$\overrightarrow{OC}=3\mathbf{a}+2\mathbf{b}+2\mathbf{c}, \qquad \overrightarrow{CA}=-\overrightarrow{OC}+\overrightarrow{OA} \text{ summed along the route.}$$

The **modulus** 模 (magnitude) of $\begin{pmatrix}x\\y\end{pmatrix}$ is its length by Pythagoras: $\sqrt{x^2+y^2}$, so $\left|\begin{pmatrix}3\\-5\end{pmatrix}\right|=\sqrt{34}\approx 5.83$. Column-vector arithmetic is componentwise: $\begin{pmatrix}2\\5\end{pmatrix}+\begin{pmatrix}-1\\3\end{pmatrix}=\begin{pmatrix}1\\8\end{pmatrix}$.

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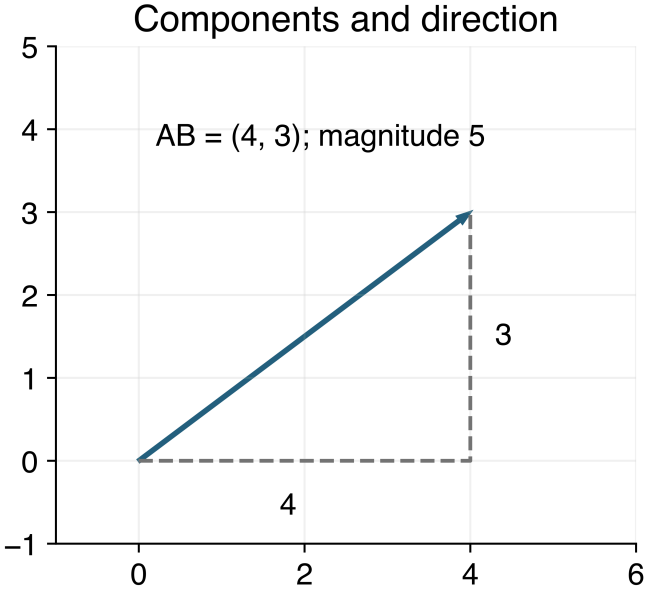


follow the route tip to tail;
the resultant joins start to end



modulus $= \sqrt{3^2 + 5^2}$
$= \sqrt{34} \approx 5.83$

Left: the route O to A to B to C with legs 3a, 2b and 2c drawn tip to tail, and the resultant OC = 3a + 2b + 2c joining start to end. Right: the column vector (3, -5) on a grid with its 3-by-5 right triangle, so its modulus is root 34, about 5.83.



AB = (4, 3) has magnitude √(4² + 3²) = 5

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Vector geometry proofs

Vector proofs show parallel or half-length relations by writing the SAME vector two ways.

> **Worked example.** $OABC$ is a **parallelogram** 平行四边形 with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$; M is the **midpoint** 中点 of diagonal OB. Prove that the **diagonals** 对角线 **bisect** 平分 each other.

Diagonal: $\overrightarrow{OB} = \mathbf{a} + \mathbf{c}$ (route $O \to A \to B$, since $\overrightarrow{AB} = \mathbf{c}$ in a parallelogram). So $\overrightarrow{OM} = \frac{1}{2}(\mathbf{a} + \mathbf{c})$. From A: $\overrightarrow{AM} = \overrightarrow{AO} + \overrightarrow{OM} = -\mathbf{a} + \frac{1}{2}(\mathbf{a} + \mathbf{c}) = \frac{1}{2}(\mathbf{c} - \mathbf{a})$. But the other diagonal has $\overrightarrow{AC} = -\mathbf{a} + \mathbf{c} = \mathbf{c} - \mathbf{a}$, so $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AC}$: M is the midpoint of AC too. Both diagonals share the midpoint – bisected. ✓

The 2HR June 2024 Q25 family asks for two things: a vector such as $\frac{8}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ in simplest form, then an unknown k for a point on a line, found by writing one vector two independent ways and equating the **coefficients** 系数 of **a** and of **b**. Here is the method in full on an original example.

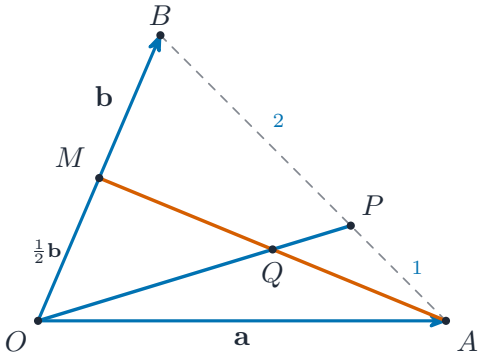
> **Worked example: finding an unknown ratio.** In triangle OAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. P lies on AB with $AP : PB = 1 : 2$, and M is the midpoint of OB. The lines OP and AM meet at Q. Find $OQ : OP$.

First, $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, so $\overrightarrow{OP} = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$. Q is on OP, so $\overrightarrow{OQ} = k\left(\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}\right)$ for some k.

Q is also on AM, where $\overrightarrow{AM} = \frac{1}{2}\mathbf{b} - \mathbf{a}$, so $\overrightarrow{OQ} = \mathbf{a} + \lambda\left(\frac{1}{2}\mathbf{b} - \mathbf{a}\right) = (1 - \lambda)\mathbf{a} + \frac{\lambda}{2}\mathbf{b}$ for some λ.

Equate the coefficients: **a**: $\frac{2}{3}k = 1 - \lambda$, and **b**: $\frac{1}{3}k = \frac{1}{2}\lambda$, so $\lambda = \frac{2}{3}k$. Then $\frac{2}{3}k = 1 - \frac{2}{3}k$, giving $k = \frac{3}{4}$ (and $\lambda = \frac{1}{2}$). So $OQ : OP = 3 : 4$, and Q is also the midpoint of AM. Never assume a point is a midpoint: give it a letter (k, λ) and equate.

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write $\overrightarrow{OQ}$ along OP and along AM,
then equate: $k = \frac{3}{4}$, $\lambda = \frac{1}{2}$

Q lies on both OP and AM, so $\overrightarrow{OQ}$ can be written two ways; equating the coefficients of **a** *and* **b** *gives $k = \frac{3}{4}$.*

Transformations: rotation, reflection, translation

A **rotation** 旋转 needs centre, angle and direction (**anticlockwise** 逆时针 positive, **clockwise** 顺时针 negative); a **reflection** 反射 needs a **mirror line** 镜像线 ($x = 1$, $y = 2$, $y = x$, $y - x = 0$); a **translation** 平移 needs a distance and direction, written as the column vector $\begin{pmatrix} a \\ b \end{pmatrix}$. Rotations, reflections and translations preserve lengths and angles, so the **image** 像 is **congruent** 全等 to the original.



Patterned tiles are built by rotating, reflecting and translating one motif

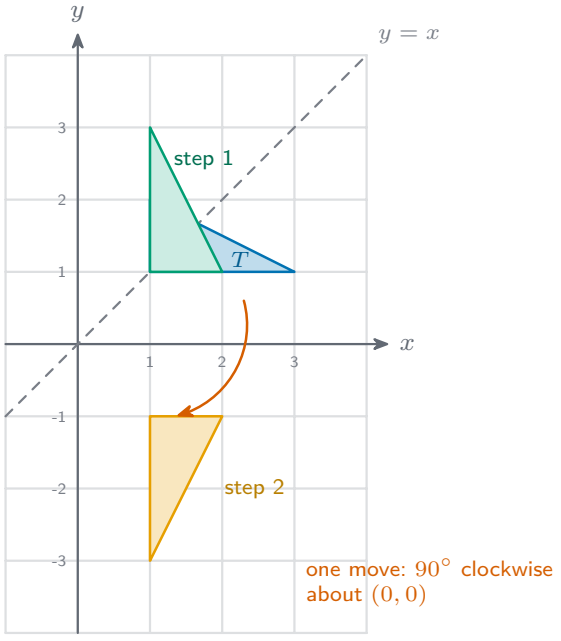
Coordinate rules for the quick cases: reflect in the x-axis: $(x, y) \to (x, -y)$; in the

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y-axis: $(x, y) \to (-x, y)$; in $y = x$: $(x, y) \to (y, x)$; rotate 90° anticlockwise about O: $(x, y) \to (-y, x)$; rotate 180°: $(x, y) \to (-x, -y)$; translate by $\begin{pmatrix} a \\ b \end{pmatrix}$: $(x, y) \to (x+a, y+b)$. Tracing paper does the rest – but the DESCRIPTION must name every required element.

> **Worked example: two transformations as one.** Triangle T has **vertices** 顶点 $(1, 1)$, $(3, 1)$ and $(1, 2)$. Reflect T in $y = x$, then reflect the image in the x-axis. Describe the single transformation that does both.

Follow a general point: $(x, y) \to (y, x)$ in $y = x$, then $(y, x) \to (y, -x)$ in the x-axis. So $(1, 1) \to (1, -1)$, $(3, 1) \to (1, -3)$ and $(1, 2) \to (2, -1)$. The rule $(x, y) \to (y, -x)$ is a **rotation of** 90° **clockwise about** $(0, 0)$. All three parts (rotation, angle with direction, centre) are needed for full marks; "turned" or "moved" earns nothing.



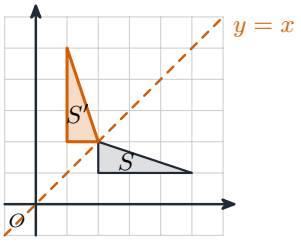
Two reflections in lines through O make one rotation: T ends up turned 90 degrees clockwise about (0, 0)

Enlargement and describing transformations

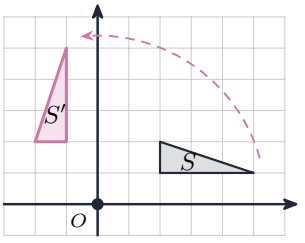
An **enlargement** 放大 is specified by a **centre of enlargement** 位似中心 and a positive **scale factor** 相似比 (fractions included, with or without the centre marked): $(x, y) \to (kx, ky)$ about the origin. Enlargements preserve angles but NOT lengths – the image is **similar** 相似, not congruent. A **complete description** names the transformation and every parameter: "rotation, 90° anticlockwise about $(0, 1)$"; "reflection

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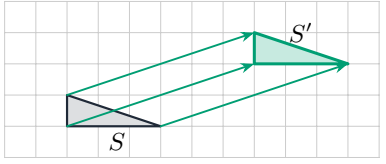
in the line $y = x$"; "translation by $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$"; "enlargement, scale factor 2, centre $(1, 1)$". Missing the centre or direction loses the mark.



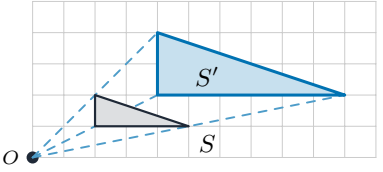
reflection in the line $y = x$



rotation $90°$ anticlockwise, centre O



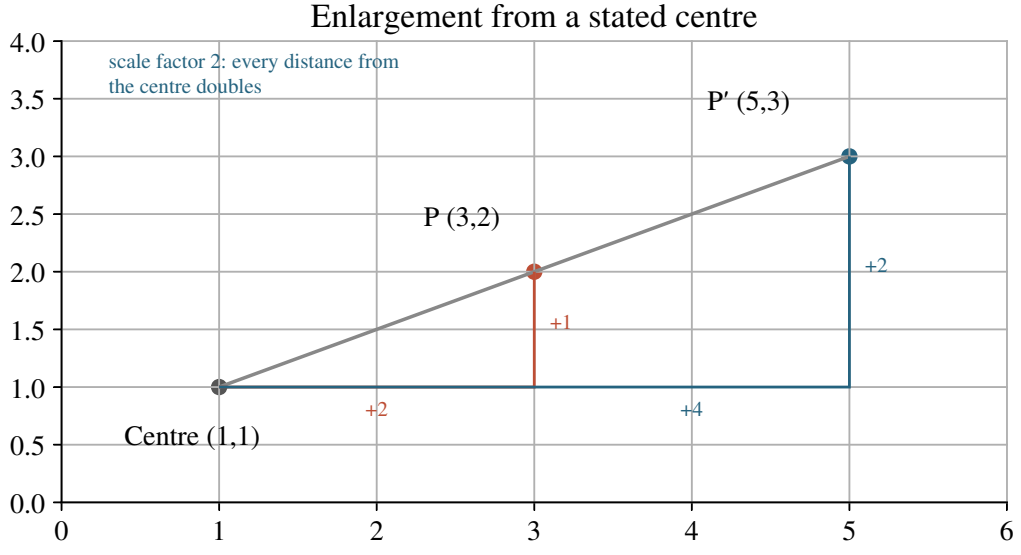
translation by $\binom{6}{2}$: every point moves 6 right, 2 up



enlargement, scale factor 2, centre O

a complete description names the transformation AND every detail: line, angle and direction, vector, or scale factor and centre

Four grids, one transformation each, showing the object S and its image S': reflection in the line $y = x$, rotation 90 degrees anticlockwise about O, translation by the column vector (6, 2), and enlargement with scale factor 2 and centre O.



Scale factor 2 from (1, 1): the image is twice as far from the centre as P

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Checklist

- Use vector notation, column vectors, scalar multiples and componentwise addition/subtraction (H 5.1 A–D).
- Find resultants along a route; reverse by negating (H 5.1 F).
- Compute moduli by Pythagoras (H 5.1 E; spec example $(3, -5) \to \sqrt{34}$).
- Prove midpoint, parallel and ratio facts by writing one vector two independent ways – never assume a midpoint (H 5.1 G; June 2024 2HR Q25).
- Rotate, reflect and translate, using column vectors for translations; state congruence preservation (F 5.2 A–I).
- Enlarge with centre and scale factor (fractions included); describe any transformation completely (F 5.2 J–M).

Boundary notes: vectors at this level are in two dimensions with positive scale factors only; negative enlargement factors belong to later study. Foundation candidates meet 5.2 only. Worked examples are adapted from the reviewed June 2024 Higher paper or original (see the meta's assessment evidence); sheet questions are original unless labelled.

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Statistics and probability

Pearson Edexcel · International GCSE · Mathematics A

4MA1 topic 6 (specification Issue 2, November 2017). Foundation 6.1–6.3 covers presenting data, averages with grouped estimates, and the full probability toolkit (scale, sample spaces, Venn diagrams, complements); Higher adds histograms with unequal classes, cumulative frequency work with median and interquartile range, and tree diagrams for independent and conditional events. On the June 2024 papers this strand supplied an unequal-class histogram (1HR Q16), a cumulative-frequency graph with quartile reads (2HR Q11) and the algebra-heavy without-replacement question (1HR Q21).

Must-know for topic 6:

Family	**Formulae**
averages	mean $= \dfrac{\sum x}{n}$; grouped mean $= \dfrac{\sum fm}{\sum f}$ (m = midpoints)
histogram	frequency density $= \dfrac{\text{frequency}}{\text{class width}}$ – bar AREA is the frequency
cumulative frequency	plot at the UPPER class boundary; median at $\frac{n}{2}$, quartiles at $\frac{n}{4}$ and $\frac{3n}{4}$
probability	$P(A') = 1 - P(A)$; $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; independent: multiply

Collecting and presenting data

Tabulate before drawing: tally into a frequency table, keep class boundaries explicit, then choose the diagram – bar chart (discrete categories), pie chart (proportions), pictogram, or a frequency polygon for grouped data. Interpretation runs both ways: read values off a given chart and compare distributions (two groups on the same axes; a truncated axis that misleads is also fair comment).

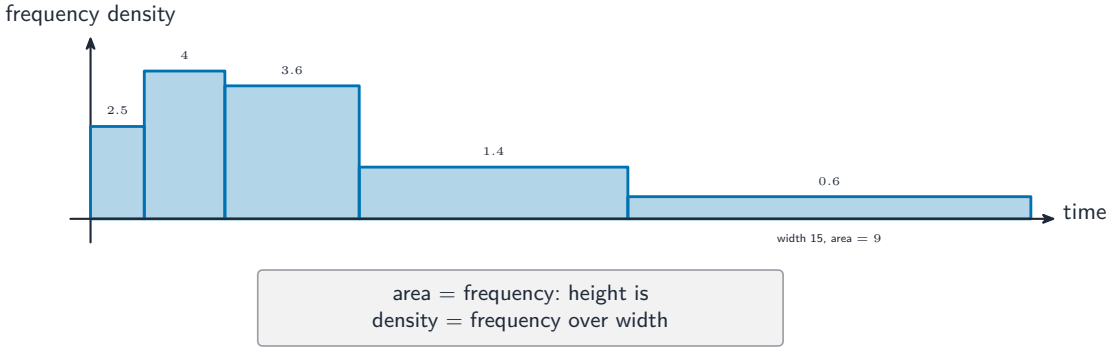
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Data is collected from a sample of people drawn from a population

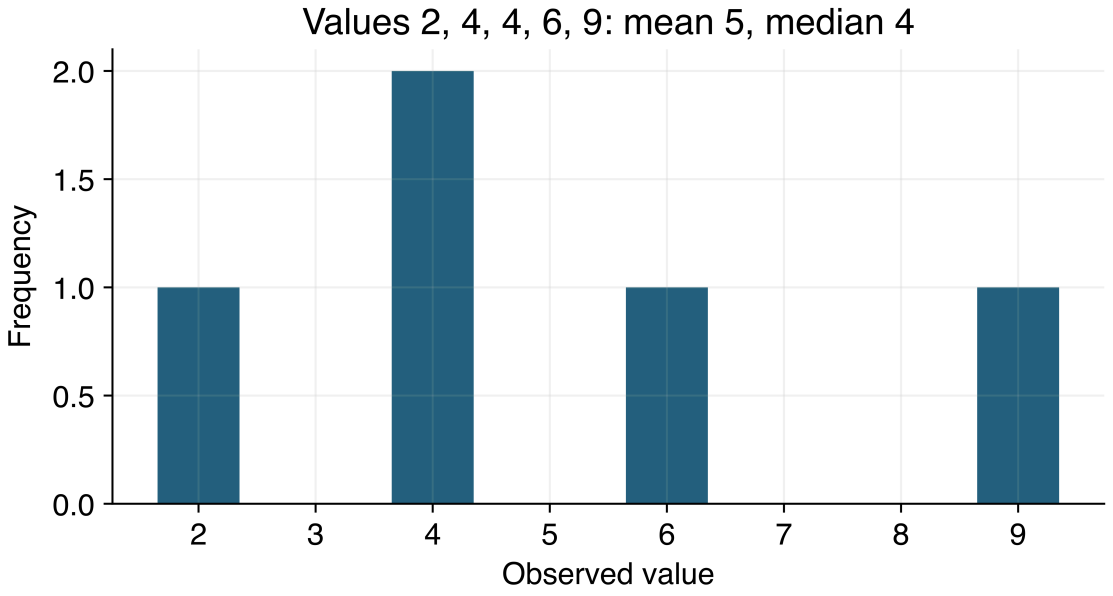
Higher draws **histograms** 直方图 with **unequal class widths** 不等组距: the bar's height is the **frequency density** 频率密度 $= \dfrac{\text{frequency}}{\text{class width}}$, so the AREA represents the frequency. Equal-width classes are the special case where height alone works.

> **Worked example: 1HR June 2024 Q16, adapted.** Classes of widths 2, 3, 5, 10, 15 hold frequencies 5, 12, 18, 14, 9. Densities: $\frac{5}{2} = 2.5$, $\frac{12}{3} = 4$, $\frac{18}{5} = 3.6$, $\frac{14}{10} = 1.4$, $\frac{9}{15} = 0.6$ – the paper's marking accepts bars of correct width whose heights are in this ratio. Check yourself: the last bar looks tall only because its width is 15; its area is $0.6 \times 15 = 9$. ✓



A histogram with unequal class widths 2, 3, 5, 10, 15, frequencies 5, 12, 18, 14, 9 and heights marked as the frequency densities 2.5, 4, 3.6, 1.4, 0.6, with the area-is-frequency takeaway.

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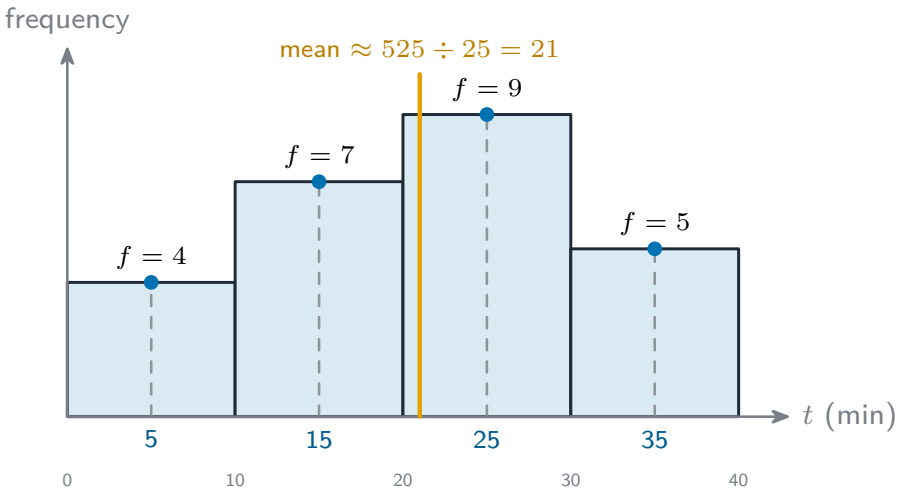


For 2, 4, 4, 6, 9: mean $= 25 \div 5 = 5$, median $= 4$

Averages and spread

The **mean** 平均数 sums and divides; the **median** 中位数 orders and takes the middle; the **mode** 众数 is the most frequent. For 3, 7, 8, 10, 12: mean 8, median 8, range $12 - 3 = 9$. For grouped data, estimate the mean with midpoints: multiply each midpoint by its frequency, sum, divide by the total; state the **modal class** 众数组 (the class with the highest frequency density when widths differ).

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midpoints (blue): each student counted as if at the middle of the class

The grouped mean treats every student as if at the midpoint of their class, so 525 divided by 25 = 21 minutes is only an estimate

Worked example: grouped mean. Times t (minutes) for 25 students:

Time	$0 < t \leq 10$	$10 < t \leq 20$	$20 < t \leq 30$	$30 < t \leq 40$
Frequency f	4	7	9	5
Midpoint m	5	15	25	35

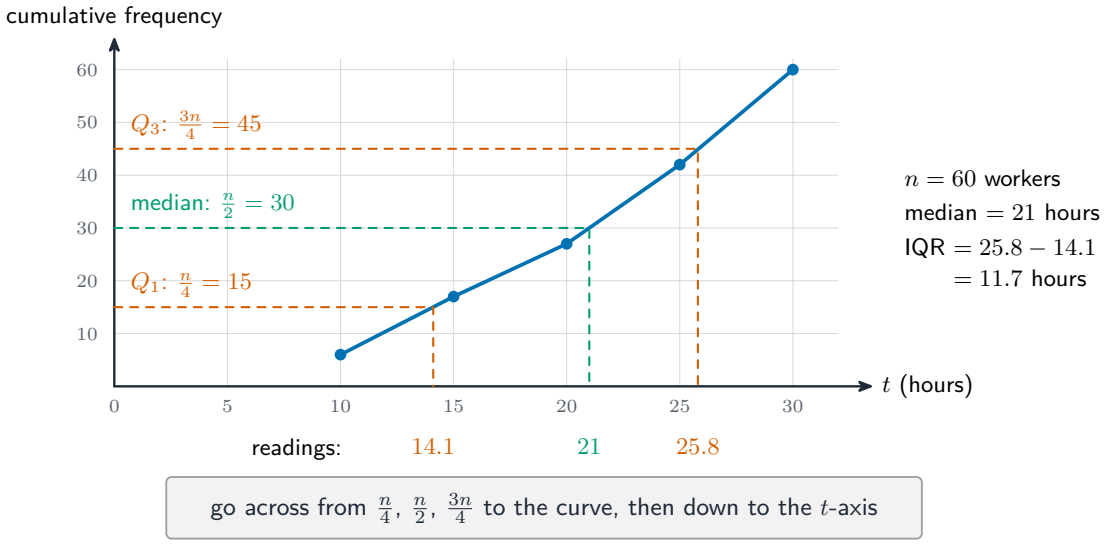
$\sum fm = 20+105+225+175 = 525$ and $\sum f = 25$, so the mean is about $525 \div 25 = 21$ minutes. It is only an **estimate**, because each student is treated as if at the midpoint. The modal class is $20 < t \leq 30$. Dividing by 4 (the number of classes) instead of 25 is the usual slip.

Higher quantifies **spread** 离散度 with the **quartiles** 四分位数: order the data, take the median, then the medians of the lower and upper halves give Q_1 and Q_3; the **interquartile range** 四分位距 is $Q_3 - Q_1$ (resistant to outliers, unlike the range). For 4, 6, 7, 9, 10, 12, 14, 15: the median is 9.5, the lower half 4, 6, 7, 9 gives $Q_1 = 6.5$, the upper half 10, 12, 14, 15 gives $Q_3 = 13$, so the IQR is $13 - 6.5 = 6.5$.

Cumulative frequency diagrams

A **cumulative frequency** 累积频率 table adds frequencies as classes pass; plot each total against its UPPER class boundary and join with a smooth increasing curve – the **ogive**. Reads: the median is the value at $\frac{n}{2}$ on the curve; the quartiles at $\frac{n}{4}$ and $\frac{3n}{4}$; the IQR is their difference; percentiles work the same way, and "how many above X" reads the total at X and subtracts from n.

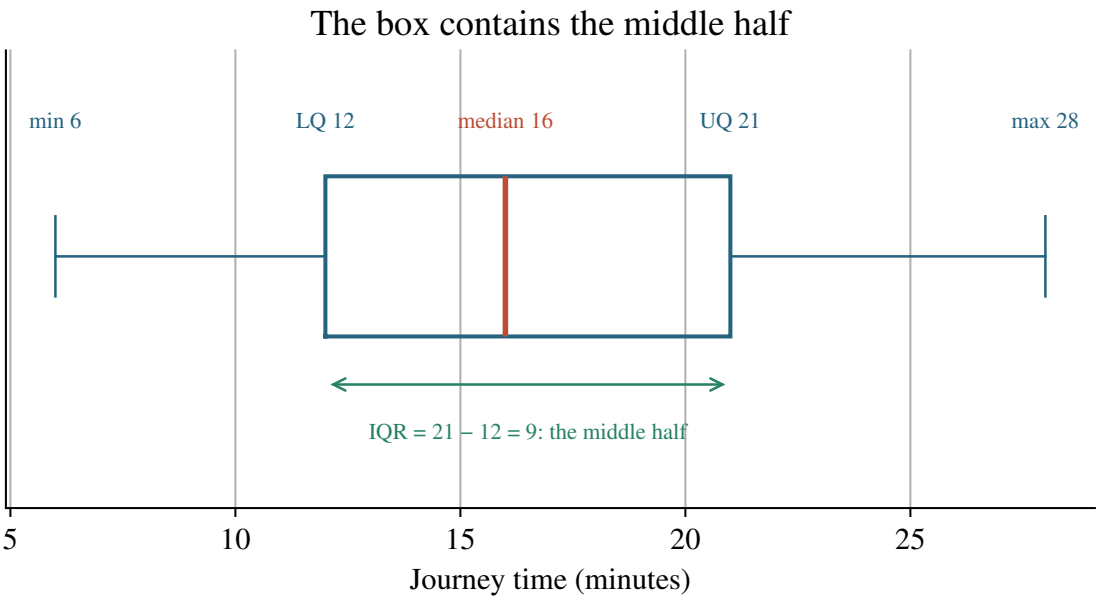
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The cumulative frequency curve for 60 workers: points (10, 6), (15, 17), (20, 27), (25, 42), (30, 60), with the median 21 and quartiles 14.1 and 25.8 marked and the IQR 11.7 stated.

Worked example: 2HR June 2024 Q11 family, adapted. 60 workers' homework times (hours per week): cumulative frequencies 6, 17, 27, 42, 60 at upper boundaries 10, 15, 20, 25, 30. The median sits at the 30th worker: between 20 (27 workers) and 25 (42), so $20 + 5 \times \frac{30-27}{42-27} = 21$ hours. The quartiles: the 15th worker gives $Q_1 = 10 + 5 \times \frac{15-6}{17-6} = 14.1$; the 45th gives $Q_3 = 25 + 5 \times \frac{45-42}{60-42} = 25.8$; IQR ≈ 11.7. Draw the reading lines on the graph – the marks are for the reads, not just the answers.

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The box holds the middle half: quartiles 12 and 21 minutes, IQR 9, median 16

Probability foundations

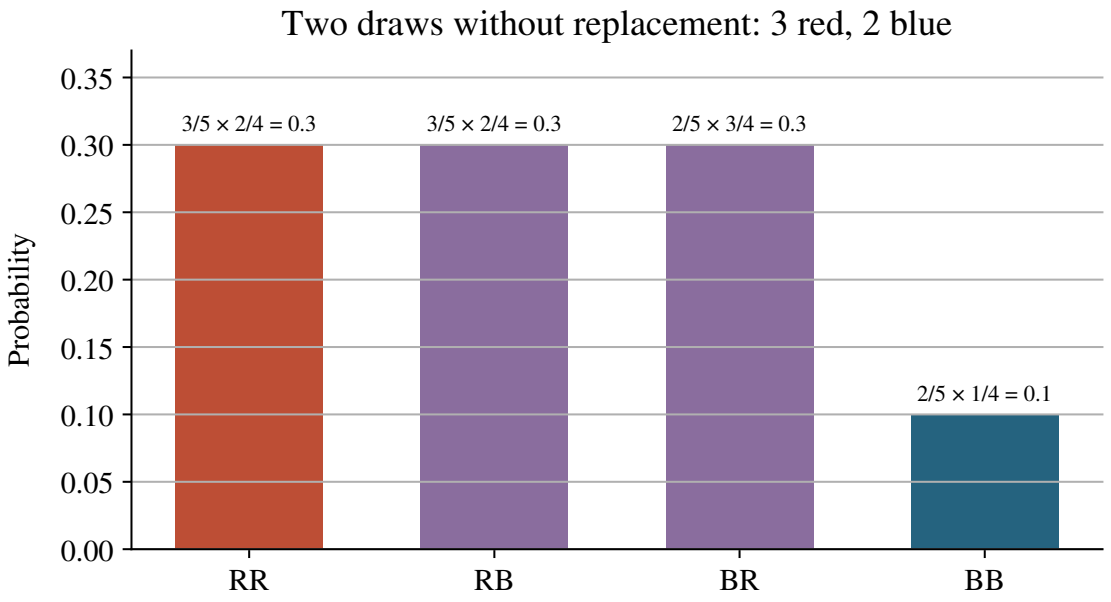
Probability measures likelihood on the scale 0 (impossible) to 1 (certain). A theoretical model counts equally likely outcomes; an experiment estimates from frequency: $P(\text{event}) \approx \frac{\text{times it happened}}{\text{trials}}$ (estimates from previously collected data included). The **sample space** 样本空间 lists all outcomes – for two successive events, a grid or systematic list; an event's probability counts the favourable cells. The **complement** 对立事件 rule: $P(A') = 1 - P(A)$ – often the fast route ("at least one" questions).



Dice: each score has probability 1/6 if the dice is fair

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Venn diagrams 维恩图: regions for each set and the overlap; probabilities fill the regions and sum to 1, with $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Example: 30 students, 18 like cricket (C), 15 tennis (T), 7 both: $P(C \cup T) = \frac{18+15-7}{30} = \frac{26}{30}$, and $P(\text{neither}) = 1 - \frac{26}{30} = \frac{2}{15}$.



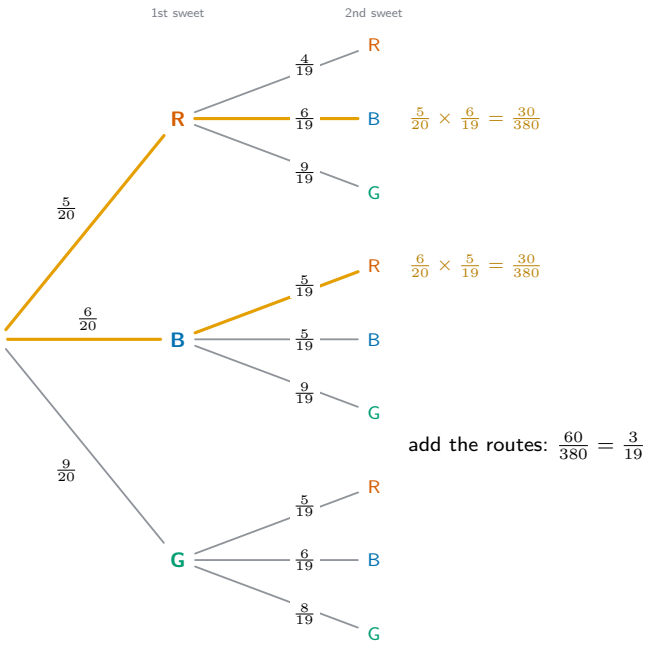
Without replacement: P(RR) = 3/5 × 2/4 = 0.3, P(BB) = 2/5 × 1/4 = 0.1

Tree diagrams and combined events

Tree diagrams 树状图 multiply along branches and add between outcomes. WITH replacement, branches repeat – the events are **independent** 独立事件: $P(A \cap B) = P(A) \times P(B)$. WITHOUT replacement, the second branch's denominator drops by one and its numerator by the first outcome: the draws are dependent, and the working shows the changing fractions.

> **Worked example: 1HR June 2024 Q21 family, adapted.** A bag holds 5 red, 6 blue and 9 green sweets; two are taken without replacement. $P(\text{one red and one blue, in either order}) = \frac{5}{20} \times \frac{6}{19} + \frac{6}{20} \times \frac{5}{19} = \frac{60}{380} = \frac{3}{19}$. The paper's version runs the same machinery with an unknown x and solves the resulting quadratic (its verified results: $x = 11$, final probability $\frac{7}{75}$): set the two-route expression equal to the given probability, expand, solve, and reject the root the context forbids.

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Without replacement the second branches are out of 19. Multiply along each route, then add the two routes: 60/380 = 3/19

Worked example: an unknown total. A bag holds n sweets, of which 6 are orange. Two are taken without replacement, and $P(\text{both orange}) = \frac{1}{3}$. Find n.

$$\frac{6}{n} \times \frac{5}{n-1} = \frac{1}{3} \;\Rightarrow\; 90 = n(n-1) \;\Rightarrow\; n^2 - n - 90 = 0 \;\Rightarrow\; (n-10)(n+9) = 0.$$

A number of sweets cannot be negative, so $n = 10$. Check: $\frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$. ✓

Conditional probability 条件概率 shrinks the world: $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$ – on a tree, the second branch given the first happened. With the bag above, $P(\text{blue second} \mid \text{red first}) = \frac{6}{19}$ read straight off the branch.

Checklist

- Tabulate and present data; interpret charts and compare distributions (F 6.1 A–C).
- Draw histograms with unequal widths via frequency density; area is the frequency (H 6.1 A; June 2024 1HR Q16).
- Mean, median, mode, range; grouped mean via midpoints; modal class (F 6.2 B–D).
- Quartiles and IQR for discrete data by position (H 6.2 B–C).
- Construct cumulative frequency curves from tables; read medians, quartiles, IQR and percentiles with drawn reading lines (H 6.1 B–C, H 6.2 A, D; June 2024 2HR Q11).

- Probability scale; theoretical vs experimental estimates; sample spaces and systematic listing for two events; Venn diagrams with the addition rule; complements (F 6.3 A–H).
- Tree diagrams: multiply along, add between; with replacement (independent) vs without; conditional probability read from branches; algebraic setups ending in quadratics (H 6.3 A–D; June 2024 1HR Q21 family).

Boundary notes: histograms use frequency density (not frequency) whenever widths differ; quartile conventions here are positional (medians of the lower and upper halves); expected value and formal set notation belong to later study. Foundation candidates meet 6.1–6.3 Foundation rows only. Worked examples are adapted from the reviewed June 2024 papers or original; sheet questions are original unless labelled.

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