

4.5 Pythagoras and trigonometry in 2D

Name: _____ Class: _____ Date: _____ Total: 25 marks

KEY WORDS sine rule • cosine rule

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Objective

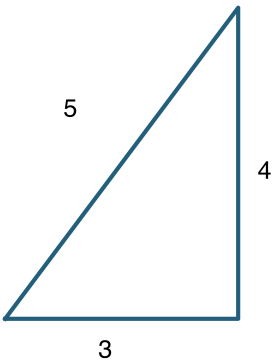
Use Pythagoras' theorem and SOH CAH TOA in right triangles, angles of elevation and depression, obtuse ratios, the sine and cosine rules and the $\frac{1}{2}ab \sin C$ area formula including in reverse (objectives Foundation 4.8 A–C and Higher 4.8 A–C, E).

1 Key knowledge and worked examples

Right triangles. $a^2 + b^2 = c^2$ (5-12-13 and 6-8-10 are triples); $\sin \theta = \text{opp/hyp}$, $\cos \theta = \text{adj/hyp}$, $\tan \theta = \text{opp/adj}$. Elevations rise from the horizontal, depressions fall; alternate angles carry them into the triangle.

Any triangle (Higher). Sine rule $a/\sin A = b/\sin B$ (a matching pair plus one part); cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$ (two sides and the included angle, or three sides); area $= \frac{1}{2}ab \sin C$, reversible. Obtuse ratios: $\sin 130^\circ = \sin 50^\circ$, $\cos 130^\circ = -\cos 50^\circ$.

Label the known sides before choosing a rule



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 A right triangle has legs 9 cm and 12 cm. Find the hypotenuse. [2]

2.2 A 6 m ladder leans against a wall at 65° to the ground. How high up the wall does it reach? [2]

2.3 Write down the value of $\sin 130^\circ$ as a ratio of an acute angle. [1]

2.4 In triangle ABC , $b = 5$, $c = 7$ and $\angle A = 60^\circ$. Find a . [3]

2.5 Triangle area is 15 cm^2 with $a = 6 \text{ cm}$ and $b = 10 \text{ cm}$. Find the included angle C . [3]

3 Exam-style questions

3.1 (*Original.*) A tower is 50 m tall. From a point 80 m from its base on level ground, find the angle of elevation of the top of the tower. [3]

3.2 (*Original.*) In triangle PQR : $PQ = 8 \text{ cm}$, $\angle P = 40^\circ$ and $\angle Q = 60^\circ$. (a) Find $\angle R$. (b) Use the sine rule to find QR . (c) Find the area of PQR . [6]

3.3 (*Original.*) A ship sails 30 km on bearing 040° from port A to a lighthouse B , then 45 km on bearing 130° to port C . (a) Show that $\angle ABC = 90^\circ$. (b) Find AC . [5]
