

6.5 Tree diagrams and combined events

Name: _____ Class: _____ Date: _____

Total: 26 marks

KEY WORDS tree diagram • conditional probability

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Objective

Draw and use tree diagrams, multiply along branches for independent events, handle without-replacement dependence, and read conditional probabilities from branches (objectives Higher 6.3 A–D).

1 Key knowledge and worked examples

Trees. Multiply along a path, add between paths. WITH replacement the second branch repeats (independent: $P(A \cap B) = P(A)P(B)$); WITHOUT replacement the second denominator drops by one and its numerator adjusts – show the changing fractions. Conditional: $P(B | A) = P(A \cap B)/P(A)$, read off the second branch.

The paper's shape. June 2024 1HR Q21 family: a bag with three colours, two draws without replacement, an unknown x – the two-route probability equals a given value, expanding gives a quadratic (the paper: $x = 11$, probability $\frac{7}{75}$). Reject the root the context forbids.

2 Practice

2.1 A coin is tossed twice. Draw or describe the tree and find $P(\text{two heads})$. [2]

2.2 A bag has 4 red and 6 blue counters; one is taken, NOT replaced, then a second. Find $P(\text{two red})$. [2]

2.3 For the same bag, find $P(\text{one of each colour})$. [3]

2.4 A spinner has $P(\text{win}) = 0.2$ each spin. Two independent spins: find $P(\text{exactly one win})$. [3]

3 Exam-style questions

3.1 (*Edexcel 4MA1 1HR June 2024 Q21 family, adapted.*) A bag holds 5 red, 6 blue and 9 green sweets. Two sweets are taken without replacement. (a) Find $P(\text{one red and one blue, in either order})$. (b) Find $P(\text{neither is green})$, and check it with the complement of “at least one green”. [5]

3.2 (*Original.*) A box contains 3 batteries of which 1 is dead. Two are tested one after another WITHOUT replacement. (a) Draw the tree. (b) Find $P(\text{the dead battery is found by the second test})$. (c) Given the first tested battery works, find the probability the second is dead. [5]

3.3 (*Original, algebra.*) A bag contains 6 mints and x toffees. Two sweets are taken without replacement and $P(\text{both are mints}) = \frac{1}{3}$. (a) Show that $x^2 + 11x - 60 = 0$. (b) Find x , rejecting any impossible root. [6]

Solutions

2.1 Four equally likely paths; $P(\text{HH}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

2.2 $\frac{4}{5} \times \frac{3}{9} = \frac{12}{45} = \frac{4}{15}$.

2.3 $\frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{4}{9} = \frac{48}{90} = \frac{8}{15}$.

2.4 $0.2 \times 0.8 + 0.8 \times 0.2 = 0.32$.

3.1 (a) $\frac{5}{20} \times \frac{6}{19} + \frac{6}{20} \times \frac{5}{19} = \frac{60}{380} = \frac{3}{19}$. (b) $P(\text{neither green}) = \frac{11}{20} \times \frac{10}{19} = \frac{110}{380} = \frac{11}{38}$, complement: $P(\text{at least one green}) = 1 - \frac{11}{38} = \frac{27}{38}$. ✓

3.2 (a) First branch: works $\frac{2}{3}$, dead $\frac{1}{3}$; second branch (2 left): after "works": works $\frac{1}{2}$, dead $\frac{1}{2}$; after "dead": works 1, dead 0. (b) $\frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times 0 = \frac{1}{3}$. (c) $P(\text{second dead} \mid \text{first works}) = \frac{1}{2}$ – the second branch after "works".

3.3 (a) $P(\text{both mints}) = \frac{6}{x+6} \times \frac{5}{x+5} = \frac{30}{(x+6)(x+5)} = \frac{1}{3}$. Cross-multiplying: $(x+6)(x+5) = 90$, so $x^2 + 11x + 30 = 90$, giving $x^2 + 11x - 60 = 0$. ✓ (b) $(x+15)(x-4) = 0$, so $x = 4$ or $x = -15$; a negative count is impossible, so $x = 4$. Check: with 4 toffees, $\frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$. ✓

Local review guidance: branch fractions adjusted for the removal; exact probabilities; algebraic parts end with a CONTEXT decision on the root. This is authored classroom guidance.