

5.3 Transformations: rotation, reflection, translation

Name: _____ Class: _____ Date: _____ Total: 15 marks

KEY WORDS rotation • reflection

Transformations: rotation, reflection, translation

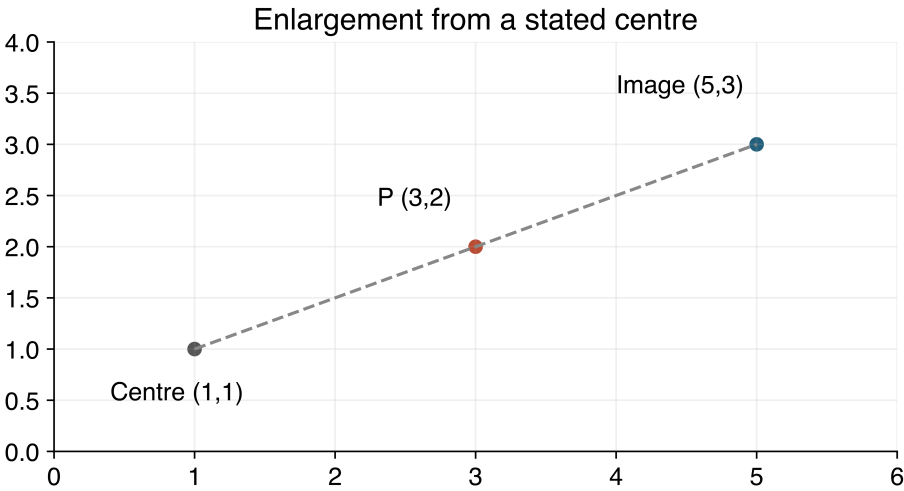
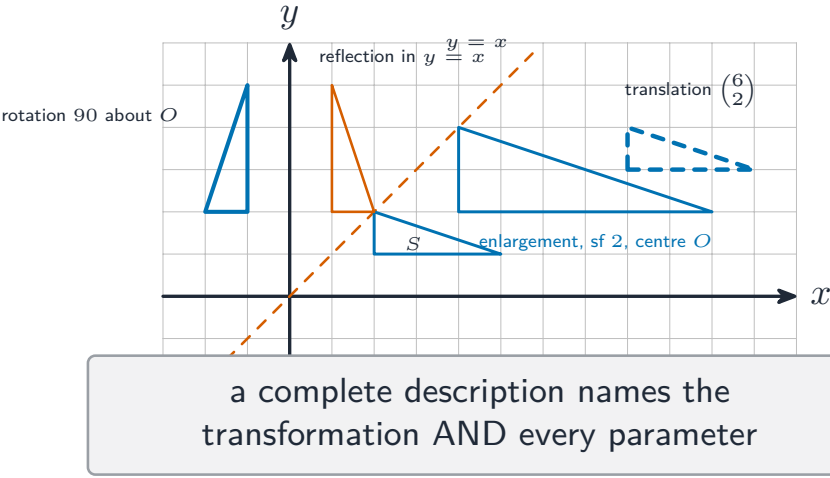
Objective

Rotate about a centre, reflect in a mirror line, translate by a column vector, and describe transformations completely; know that these three preserve length and angle so images are congruent (objectives Foundation 5.2 A–I).

1 Key knowledge and worked examples

The three isometries. Rotation: centre + angle + direction (anticlockwise positive). Reflection: a mirror line such as $x = 1$, $y = 2$, $y = x$. Translation: distance and direction as the column vector $\begin{pmatrix} a \\ b \end{pmatrix}$. All three preserve lengths and angles, so every image is congruent to the original.

Coordinate shortcuts. Reflect: x -axis $(x, y) \rightarrow (x, -y)$; y -axis $(x, y) \rightarrow (-x, y)$; $y = x$: $(x, y) \rightarrow (y, x)$. Rotate about O : 90° anticlockwise $(x, y) \rightarrow (-y, x)$; 180° : $(x, y) \rightarrow (-x, -y)$. Translate: $(x, y) \rightarrow (x + a, y + b)$.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Reflect (3,5) in the x -axis. [1]

2.2 Reflect (3,5) in the line $y = x$. [1]

2.3 Rotate $(-2, 4)$ through 90° anticlockwise about O . [2]

2.4 Translate $(1, -3)$ by the vector $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$ [1]

2.5 A shape and its image have equal side lengths and equal angles, and every point has moved the same distance in the same direction. Name the transformation. [1]

3 Exam-style questions

3.1 (*Edexcel 4MA1 2HR June 2024 Q23 family, adapted.*) The point $P(2, 7)$ lies on the graph of $y = f(x)$. Write down the coordinates of the image of P on the graph of (a) $y = f(x - 4)$ (b) $y = 2f(x)$. [2]

3.2 (*Original.*) Triangle A has vertices $(1, 1)$, $(4, 1)$, $(1, 3)$. It is rotated 90° clockwise about $(0, 0)$. (a) Draw or describe the image's vertices. (b) Describe fully the single transformation that maps the image back onto A . [4]

3.3 (*Original.*) Shape B is the image of shape A under a reflection in the line $x = 2$. State the coordinates of the image of the point $(5, -1)$, and explain why the image is congruent to A . [3]

Solutions

2.1 $(3, -5)$.

2.2 $(5, 3)$.

2.3 $(-4, -2)$.

2.4 $(5, 3)$.

2.5 A translation.

3.1 (a) $y = f(x - 4)$ shifts 4 right: $(6, 7)$. (b) $y = 2f(x)$ stretches vertically, factor 2: $(2, 14)$.

3.2 (a) 90° clockwise about O : $(x, y) \rightarrow (y, -x)$, giving $(1, -1)$, $(1, -4)$, $(3, -1)$. (b) Rotation, 90° anticlockwise about $(0, 0)$ (the inverse).

3.3 Reflection in $x = 2$: x -coordinates satisfy $\frac{x+x'}{2} = 2$, so $x' = 4 - 5 = -1$: the image is $(-1, -1)$. Reflections preserve lengths and angles, so the image is congruent to A .

Local review guidance: the correct rule or construction applied; exact coordinates; descriptions name centre/line/vector AND direction/factor. This is authored classroom guidance.