4.8 Similarity ratios and mixed practice

Name: _____ Class: _____ Date: _____ **Total: 29 marks**

KEY WORDS scale factor • similar

Similarity ratios and mixed practice

Objective

Use length : area : volume ratios $k : k^2 : k^3$ in both directions, on similar figures and solids, with maps and models, alongside mixed geometry and trigonometry chains (objectives Foundation 4.11 and Higher 4.11 A–C).

1 Key knowledge and worked examples

> **The ratio ladder.** Similar figures: corresponding lengths scale by k, areas by k^2, volumes by k^3. Read backwards: an area ratio $4 : 9$ gives a length ratio $2 : 3$; a volume ratio $64 : 1$ gives $4 : 1$ for lengths. June 2024 2HR Q24's similar cone-plus-hemisphere solids gave $k^3 = 125/8$, so $k = 2.5$.

2 Practice

2.1 Two similar jugs have height ratio $2 : 5$. Write down their volume ratio. [2]

2.2 Two similar photographs have areas 18 cm^2 and 50 cm^2. Find the ratio of their widths. [3]

2.3 On a $1 : 25\,000$ map a road is 6 cm. Find the real length in km. [2]

2.4 A model car is built at $1 : 48$. The real car is 4.2 m long. Find the model length in cm. [3]

2.5 Similar cones have base radii 3 cm and 6 cm. The smaller has volume 12π cm^3. Find the larger's volume. [3]

3 Exam-style questions

3.1 *(Edexcel 4MA1 2HR June 2024 Q24 family, adapted.)* Solid B is similar to solid A, with every length of B equal to k times the corresponding length of A. The volumes satisfy $V_B = \frac{125}{8} V_A$. (a) Find k. (b) A third solid C has every length 4 times solid A's. Write down $V_C : V_A$. [4]

3.2 *(Original.)* Two similar tins of soup have heights 8 cm and 12 cm. The larger tin holds 486 cm^3. (a) Find the volume of the smaller tin. (b) The larger tin's label wraps its curved surface, area 150 cm^2. Find the label area of the smaller tin. [6]

3.3 *(Original, mixed transfer.)* A rectangular paddock measures 40 m by 25 m on a plan drawn at 1 : 1000. (a) Find the plan's dimensions in cm. (b) A scale model of a prize cow is 1 : 20; the real cow has volume-equivalent body mass 640 kg and the model uses material of the same density. Find the model's mass in grams. [6]

Solutions

2.1 $2^3 : 5^3 = 8 : 125$.

2.2 $\sqrt{18} : \sqrt{50} = 3\sqrt{2} : 5\sqrt{2} = 3 : 5$.

2.3 $6 \times 25\,000 = 150\,000$ cm $= 1.5$ km.

2.4 4.2 m = 420 cm; $420/48 = 8.75$ cm.

2.5 $k = 2$, so volumes scale by 8: 96π cm^3.

3.1 (a) $k^3 = 125/8$, so $k = \sqrt[3]{125/8} = 5/2 = 2.5$. (b) $4^3 = 64$, so $V_C : V_A = 64 : 1$.

3.2 (a) $k = 8/12 = 2/3$; volume factor $k^3 = 8/27$; smaller $= 486 \times 8/27 = 144$ cm^3. (b) Area factor $k^2 = 4/9$; smaller label $= 150 \times 4/9 = 66.7$ cm^2 (3 s.f.).

3.3 (a) 40 m = 4000 cm at 1 : 1000 → 4 cm; 25 m → 2.5 cm. (b) Volume factor $(1/20)^3 = 1/8000$; mass $= 640/8000 = 0.08$ kg $= 80$ g.

Local review guidance: the ladder direction stated (length → area → volume) before multiplying; exact ratios simplified; scale answers carried through the unit conversions. This is authored classroom guidance.