

4.7 Mensuration: perimeter, area, surface and volume

Name: _____ Class: _____ Date: _____ Total: 28 marks

KEY WORDS sector • volume

Mensuration: perimeter, area, surface and volume

Objective

Find perimeters and areas of composite shapes, circles and sectors; surface areas and volumes of prisms, cylinders, cones and spheres; convert metric units (objectives Foundation 4.9–4.10 and Higher 4.9, 4.10).

1 Key knowledge and worked examples

Area toolkit. Triangle $\frac{1}{2}bh$, parallelogram bh , trapezium $\frac{1}{2}(a + b)h$, circle πr^2 with circumference $2\pi r$. Higher sectors: arc $= \frac{\theta}{360} \times 2\pi r$, area $= \frac{\theta}{360} \times \pi r^2$ – for $r = 9$, $\theta = 40^\circ$: arc $2\pi \approx 6.28$, area $9\pi \approx 28.3$. Composite shapes split into parts before any formula is applied.

Solids. Prism = cross-section $\times$ length; cylinder $V = \pi r^2 h$, surface $2\pi r^2 + 2\pi rh$; cone $V = \frac{1}{3}\pi r^2 h$, curved surface πrl with l the slant; sphere $V = \frac{4}{3}\pi r^3$, surface $4\pi r^2$. Unit chains: area 100^2 , volume 100^3 between cm and m.

2 Practice

- 2.1 Find the area of a trapezium with parallel sides 7 cm and 11 cm and height 4 cm.

[2]
- 2.2 Find the circumference and area of a circle of radius 5 cm, exact in terms of π .

[2]

- 2.3 A sector has $r = 9$ cm and $\theta = 40^\circ$. Find its arc length.

[2]
- 2.4 Find the volume of a cylinder with $r = 5$ cm and $h = 10$ cm, in terms of π .

[2]
- 2.5 A cone has $r = 3$ cm and $h = 4$ cm. Find its slant, volume and curved surface area, in terms of π where exact.

[4]
- ### 3 Exam-style questions

3.1 (*Edexcel 4MA1 2HR June 2024 Q10, adapted.*) Hexagon $ABCDEF$ has $AB \parallel FC \parallel ED$ with $AB = 11$ cm, $ED = FC = 23$ cm, the perpendicular distance between FC and ED equal to 4.7 cm, and $BC = 5$ cm making a 30° angle with FC . Find the area of the hexagon.

[5]

3.2 (*Original.*) A sphere has radius 6 cm. (a) Find its volume in terms of π . (b) Find its surface area in terms of π . (c) A cylinder has the same radius and the same volume; find its height.

[5]

3.3 (*Original.*) A barn silo is a cylinder ($r = 2$ m, $h = 5$ m) topped by a hemisphere. (a) Find the total volume in terms of π . (b) Find the total surface area in terms of π . [6]

Solutions

2.1 $\frac{1}{2}(7 + 11) \times 4 = 36 \text{ cm}^2$.

2.2 $10\pi \text{ cm}$; $25\pi \text{ cm}^2$.

2.3 $\frac{40}{360} \times 2\pi \times 9 = 2\pi \approx 6.28 \text{ cm}$.

2.4 $\pi \times 25 \times 10 = 250\pi \text{ cm}^3$.

2.5 Slant $l = \sqrt{9 + 16} = 5 \text{ cm}$; $V = \frac{1}{3}\pi \times 9 \times 4 = 12\pi \text{ cm}^3$; curved surface $\pi \times 3 \times 5 = 15\pi \text{ cm}^2$.

3.1 The strip between the parallels FC and ED is a rectangle 23×4.7 ; above it, the height of the trapezium $ABCF$ is $5 \sin 30^\circ = 2.5 \text{ cm}$. Split: rectangle $23 \times 4.7 = 108.1$ plus trapezium $\frac{1}{2}(11 + 23) \times 2.5 = 42.5$; total 150.6 cm^2 (exact $\frac{753}{5}$).

3.2 (a) $\frac{4}{3}\pi \times 216 = 288\pi \text{ cm}^3$. (b) $4\pi \times 36 = 144\pi \text{ cm}^2$. (c) $\pi \times 36 \times h = 288\pi$ gives $h = 8 \text{ cm}$.

3.3 (a) Cylinder $\pi \times 4 \times 5 = 20\pi$; hemisphere $\frac{2}{3}\pi \times 8 = \frac{16}{3}\pi$; total $\frac{76}{3}\pi \text{ m}^3$. (b) Cylinder wall $2\pi \times 2 \times 5 = 20\pi$; hemisphere curved $2\pi \times 4 = 8\pi$; total $28\pi \text{ m}^2$.

Local review guidance: the split drawn or named before formulas; exact π forms kept until the final line; units squared or cubed to match. This is authored classroom guidance.