4.6 Three-dimensional trigonometry

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS space diagonal

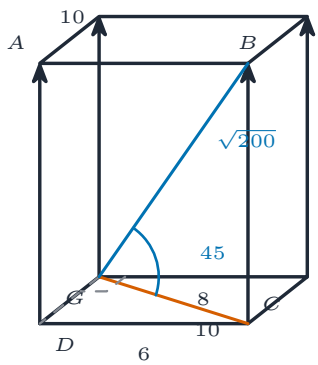
Three-dimensional trigonometry

Objective

Extract right triangles from cuboids and other solids, use Pythagoras in 3D for face and space diagonals, and finish with trigonometry or the cosine rule on the extracted triangle (objectives Higher 4.8 D and F).

1 Key knowledge and worked examples

The 3D method. A 3D problem is a 2D problem in disguise: name the triangle the question asks about, find its sides one at a time (each by its own right-triangle Pythagoras or trigonometry), then apply Pythagoras, SOH CAH TOA or the cosine rule to that triangle. In a $6 \times 8 \times 10$ cuboid the base diagonal is 10 and the space diagonal is $\sqrt{200} \approx 14.1$.



$$\text{base } 6^2 + 8^2 = 10^2; \text{ space}$$
$$10^2 + 10^2 = 200; \tan\theta = \frac{10}{10}$$

2 Practice

2.1 A cuboid has dimensions 3 cm $\times$ 4 cm $\times$ 12 cm. Find the length of its space diagonal. [3]

2.2 For the cuboid above, find the length of a diagonal of the 3 cm $\times$ 4 cm face. [2]

2.3 In the $6 \times 8 \times 10$ cuboid, find the angle between the space diagonal and the base. [3]

2.4 A square-based pyramid has base edge 10 cm and all slant edges 13 cm. Find the height of the apex above the centre of the base. [3]

3 Exam-style questions

3.1 *(Edexcel 4MA1 1HR June 2024 Q22, adapted.)* The points B, M and P lie in a solid so that $BM = \sqrt{200}$, $BP = \sqrt{244}$ and $MP = 10$ (each length itself found by Pythagoras in a right triangle of the solid). Find $\angle BMP$. [4]

3.2 *(Original.)* A cuboid has base 5 cm $\times$ 12 cm and height 8 cm. G is a bottom corner and B the opposite top corner; C is the base corner diagonally opposite G. (a) Find GC. (b) Find GB. (c) Find the angle between GB and the base diagonal GC. [6]

3.3 *(Original.)* A vertical flagpole of height 12 m stands at the centre O of a horizontal square courtyard of side 16 m. P is a corner of the courtyard. Find the angle of elevation of the top of the flagpole from P. [5]

Solutions

2.1 $\sqrt{9+16+144} = \sqrt{169} = 13$ cm.

2.2 $\sqrt{9+16} = 5$ cm.

2.3 The space diagonal meets the base at the end of the base diagonal, so $\tan\theta = 10/10 = 1$ and $\theta = 45°$.

2.4 Half the base diagonal is $\frac{1}{2}\sqrt{200} = \sqrt{50}$; height $= \sqrt{169-50} = \sqrt{119} = 10.9$ cm (3 s.f.).

3.1 Cosine rule at M: $\cos BMP = \dfrac{200+100-244}{2\sqrt{200}\sqrt{100}} = \dfrac{56}{282.8} = 0.198$, so $\angle BMP = 78.6°$.

3.2 (a) $GC = \sqrt{25+144} = 13$ cm. (b) $GB = \sqrt{25+144+64} = \sqrt{233} = 15.3$ cm (3 s.f.). (c) $\tan\theta = 8/13$, so $\theta = 31.6°$ (1 d.p.).

3.3 OP is half the base diagonal: $\frac{1}{2} \times 16\sqrt{2} = 11.31$ m. $\tan\theta = 12/11.31 = 1.061$, so $\theta = 46.7°$ (1 d.p.).

Local review guidance: the extracted triangle identified before any computation; surds or 3 s.f. as asked; the angle's vertex named in the notation. This is authored classroom guidance.