4.5 Pythagoras and trigonometry in 2D

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS sine rule • cosine rule

Pythagoras and trigonometry in 2D

Objective

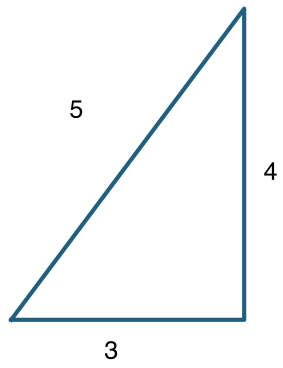
Use Pythagoras' theorem and SOH CAH TOA in right triangles, angles of elevation and depression, obtuse ratios, the sine and cosine rules and the ½ab sin C area formula including in reverse (objectives Foundation 4.8 A–C and Higher 4.8 A–C, E).

1 Key knowledge and worked examples

Right triangles. $a^2 + b^2 = c^2$ (5-12-13 and 6-8-10 are triples); $\sin\theta = \text{opp/hyp}$, $\cos\theta = \text{adj/hyp}$, $\tan\theta = \text{opp/adj}$. Elevations rise from the horizontal, depressions fall; alternate angles carry them into the triangle.

Any triangle (Higher). Sine rule $a/\sin A = b/\sin B$ (a matching pair plus one part); cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$ (two sides and the included angle, or three sides); area $= \frac{1}{2}ab\sin C$, reversible. Obtuse ratios: $\sin 130° = \sin 50°$, $\cos 130° = -\cos 50°$.

Label the known sides before choosing a rule



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 A right triangle has legs 9 cm and 12 cm. Find the hypotenuse. [2]

2.2 A 6 m ladder leans against a wall at 65° to the ground. How high up the wall does it reach? [2]

2.3 Write down the value of $\sin 130°$ as a ratio of an acute angle. [1]

2.4 In triangle ABC, $b = 5$, $c = 7$ and $\angle A = 60°$. Find a. [3]

2.5 Triangle area is 15 cm^2 with $a = 6$ cm and $b = 10$ cm. Find the included angle C. [3]

3 Exam-style questions

3.1 *(Original.)* A tower is 50 m tall. From a point 80 m from its base on level ground, find the angle of elevation of the top of the tower. [3]

3.2 *(Original.)* In triangle PQR: $PQ = 8$ cm, $\angle P = 40°$ and $\angle Q = 60°$. (a) Find $\angle R$. (b) Use the sine rule to find QR. (c) Find the area of PQR. [6]

3.3 *(Original.)* A ship sails 30 km on bearing 040° from port A to a lighthouse B, then 45 km on bearing 130° to port C. (a) Show that $\angle ABC = 90°$. (b) Find AC. [5]

Solutions

2.1 $\sqrt{81+144} = \sqrt{225} = 15$ cm.

2.2 $6\sin 65^\circ = 5.44$ m (3 s.f.).

2.3 $\sin 50^\circ$.

2.4 $a^2 = 25 + 49 - 70\cos 60^\circ = 74 - 35 = 39$, so $a = 6.24$ (3 s.f.).

2.5 $\frac{1}{2} \times 6 \times 10 \times \sin C = 15$ gives $\sin C = 0.5$, so $C = 30^\circ$.

3.1 $\tan\theta = 50/80 = 0.625$, so $\theta = 32.0^\circ$ (1 d.p.).

3.2 (a) $\angle R = 180 - 40 - 60 = 80^\circ$. (b) QR is opposite $\angle P$: $QR/\sin 40 = 8/\sin 80$, so $QR = 8\sin 40/\sin 80 = 5.22$ cm (3 s.f.). (c) Using sides $PQ = 8$ and $QR = 5.22$ with included angle $Q = 60^\circ$: area $= \frac{1}{2} \times 8 \times 5.22 \times \sin 60 = 18.1$ cm^2 (3 s.f.).

3.3 (a) At B, the back-bearing of A is $040 + 180 = 220^\circ$; C bears 130°, so $\angle ABC = 220 - 130 = 90^\circ$. (b) $AC = \sqrt{30^2 + 45^2} = \sqrt{2925} = 54.1$ km (3 s.f.).

Local review guidance: the correct rule chosen for the given parts; 3 s.f. accuracy with units; bearings angles found via back-bearings, not guesses. This is authored classroom guidance.