

4.5 Pythagoras and trigonometry in 2D

Name: _____ Class: _____ Date: _____

Total: 25 marks

KEY WORDS sine rule • cosine rule

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Objective

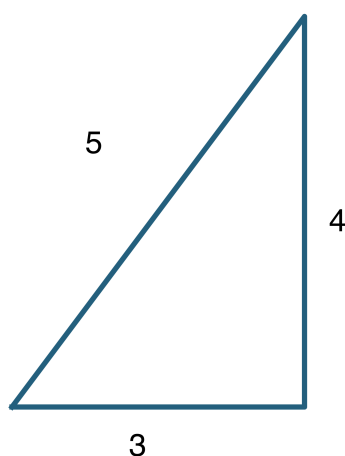
Use Pythagoras' theorem and SOH CAH TOA in right triangles, angles of elevation and depression, obtuse ratios, the sine and cosine rules and the $\frac{1}{2}ab \sin C$ area formula including in reverse (objectives Foundation 4.8 A–C and Higher 4.8 A–C, E).

1 Key knowledge and worked examples

Right triangles. $a^2 + b^2 = c^2$ (5-12-13 and 6-8-10 are triples); $\sin \theta = \text{opp/hyp}$, $\cos \theta = \text{adj/hyp}$, $\tan \theta = \text{opp/adj}$. Elevations rise from the horizontal, depressions fall; alternate angles carry them into the triangle.

Any triangle (Higher). Sine rule $a/\sin A = b/\sin B$ (a matching pair plus one part); cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$ (two sides and the included angle, or three sides); area $= \frac{1}{2}ab \sin C$, reversible. Obtuse ratios: $\sin 130^\circ = \sin 50^\circ$, $\cos 130^\circ = -\cos 50^\circ$.

Label the known sides before choosing a rule



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 A right triangle has legs 9 cm and 12 cm. Find the hypotenuse. [2]

2.2 A 6 m ladder leans against a wall at 65° to the ground. How high up the wall does it reach? [2]

2.3 Write down the value of $\sin 130^\circ$ as a ratio of an acute angle. [1]

2.4 In triangle ABC , $b = 5$, $c = 7$ and $\angle A = 60^\circ$. Find a . [3]

2.5 Triangle area is 15 cm^2 with $a = 6 \text{ cm}$ and $b = 10 \text{ cm}$. Find the included angle C . [3]

3 Exam-style questions

3.1 (*Original.*) A tower is 50 m tall. From a point 80 m from its base on level ground, find the angle of elevation of the top of the tower. [3]

3.2 (*Original.*) In triangle PQR : $PQ = 8 \text{ cm}$, $\angle P = 40^\circ$ and $\angle Q = 60^\circ$. (a) Find $\angle R$. (b) Use the sine rule to find QR . (c) Find the area of PQR . [6]

3.3 (*Original.*) A ship sails 30 km on bearing 040° from port A to a lighthouse B , then 45 km on bearing 130° to port C . (a) Show that $\angle ABC = 90^\circ$. (b) Find AC . [5]

Solutions

2.1 $\sqrt{81 + 144} = \sqrt{225} = 15$ cm.

2.2 $6 \sin 65^\circ = 5.44$ m (3 s.f.).

2.3 $\sin 50^\circ$.

2.4 $a^2 = 25 + 49 - 70 \cos 60^\circ = 74 - 35 = 39$, so $a = 6.24$ (3 s.f.).

2.5 $\frac{1}{2} \times 6 \times 10 \times \sin C = 15$ gives $\sin C = 0.5$, so $C = 30^\circ$.

3.1 $\tan \theta = 50/80 = 0.625$, so $\theta = 32.0^\circ$ (1 d.p.).

3.2 (a) $\angle R = 180 - 40 - 60 = 80^\circ$. (b) QR is opposite $\angle P$: $QR/\sin 40 = 8/\sin 80$, so $QR = 8 \sin 40/\sin 80 = 5.22$ cm (3 s.f.). (c) Using sides $PQ = 8$ and $QR = 5.22$ with included angle $Q = 60^\circ$: area $= \frac{1}{2} \times 8 \times 5.22 \times \sin 60 = 18.1$ cm² (3 s.f.).

3.3 (a) At B , the back-bearing of A is $040 + 180 = 220^\circ$; C bears 130° , so $\angle ABC = 220 - 130 = 90^\circ$. (b) $AC = \sqrt{30^2 + 45^2} = \sqrt{2925} = 54.1$ km (3 s.f.).

Local review guidance: the correct rule chosen for the given parts; 3 s.f. accuracy with units; bearings angles found via back-bearings, not guesses. This is authored classroom guidance.