

3.1 Sequences

Name: _____ Class: _____ Date: _____

KEY WORDS common difference

<!-- Session 2 native-skill companion -->

Objective

Practise sequences within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Arithmetic sequences and n th terms · Foundation

Find a constant difference for an arithmetic sequence. Its n th term is $a + (n-1)d$. A term-to-term rule describes how to reach the next term; a position-to-term rule gives a term directly.

$$u_n = a + (n - 1)d$$

For 5, 8, 11, 14, ... the common difference is 3. The n th term is $5 + 3(n-1) = 3n + 2$. At $n=8$, $u_8 = 26$. To find the position of 62, solve $3n + 2 = 62$, giving $n=20$.

The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge.

Generate several terms and check a proposed n th-term rule. Infinite geometric series and advanced sum formulae are excluded from this Foundation/Core lesson.

■ Arithmetic sequences and finite sums · Higher

For an arithmetic sequence, $u_n = a + (n-1)d$. Pairing the first and last terms gives equal pair sums $a + u_n$. Therefore the first n terms have sum $S_n = n(a + u_n)/2 = n[2a + (n-1)d]/2$. A sequence lists terms; a series adds them. State the first index before using a formula.

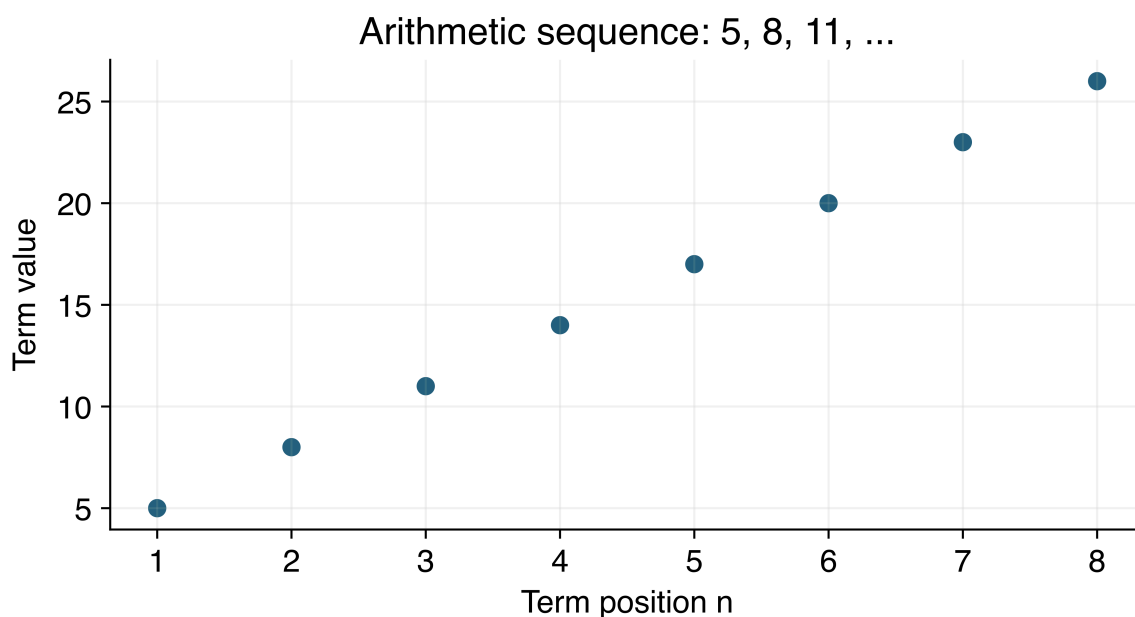
$$u_n = a + (n - 1)d, \quad S_n = \frac{n}{2} [2a + (n - 1)d]$$

For $a=5, d=3, n=8$, $u_8 = 5 + 7 \times 3 = 26$ and $S_8 = 8(5 + 26)/2 = 124$. If the second term is 7 and the fifth is 19, then $3d = 19 - 7 = 12$, so $d=4$ and $a = 7 - 4 = 3$. The first four terms

3,7,11,15 sum to 36, also $4(3+15)/2=36$. The sum formula depends on a constant difference.

A term is not a sum. The first term is indexed by 1 in these formulae. An arithmetic sum requires a constant difference; increasing terms alone do not establish this.

4MA1 Higher 3.1 A–C covers common difference, the arithmetic n th term and the sum of the first n arithmetic terms. Geometric finite/infinite sum formulae are excluded from this supported focus lesson.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Foundation) For $a=5, d=3$, find u_8 .

2.2 (Foundation) At what position does 62 occur in $u_n=3n+2$?

2.3 (Foundation) Find u_4 for $u_n = 3n + 2$.

2.4 (Higher) Find the sum of the first 8 terms with $a=5, d=3$.

2.5 (Higher) Find the sum of the first four arithmetic terms 3, 7, 11, 15.

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: The first term of a sequence always has index zero.

Solutions

2.1 $u_8 = 5 + (8-1) \times 3 = 26.$

2.2 Solve $3n+2=62$: $3n=60$, hence $n=20$.

2.3 $u_4 = 3 \times 4 + 2 = 14.$

2.4 $S_8 = 8[2 \times 5 + 7 \times 3]/2 = 124.$

2.5 $S_4 = 4(3+15)/2 = 36.$

3.1 False. The convention $a_1=5$, $a_2=8$, $a_3=11$ indexes its first term by one. An initial value a_0 is possible only when explicitly specified.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.