

Exercise walk-through

6.6

Mixed practice: statistics and probability

Pearson Edexcel · International GCSE · Mathematics A

1. Topic 6 at a glance

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Foundation 6.1-6.3: presenting data, averages with grouped estimates, and the probability toolkit. Higher adds unequal-class histograms, cumulative frequency with quartiles, and tree diagrams for independent and conditional events.

Must-know: grouped mean via midpoints; density is frequency over width, AREA is frequency; ogive reads at $n/2$, $n/4$, $3n/4$; $P(A') = 1 - P(A)$; $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$; trees multiply along, add between.

5. Probability foundations

Probability foundations

Scale 0 to 1; theoretical counts equally likely outcomes, experimental divides successes by trials. Sample spaces list outcomes - grids for two events; complements turn 'at least one' into one minus none.

Venn regions sum to the total: 30 students, 18 cricket, 15 tennis, 7 both gives $P(\text{neither}) = 1 - 26/30 = 2/15$, through $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

6. Trees and combined events

Trees and combined events

Multiply along a path, add between paths. With replacement the branches repeat (independent); without replacement the second denominator drops by one and the numerator adjusts.

June 2024 1HR Q21 family: 5 red, 6 blue, 9 green, two draws without replacement - one of each of red/blue either way is $\frac{5}{20} \times \frac{6}{19} + \frac{6}{20} \times \frac{5}{19} = \frac{3}{19}$. The paper's unknown-x version ends in a quadratic ($x = 11$, probability $\frac{7}{75}$). Conditional reads off the branch: $P(\text{blue second given red first}) = \frac{6}{19}$.

Exercises 2.1·2.3

2.1 A histogram bar of width 4 has density 3.5. Find its frequency. [2]

2.2 Find the mean of 6, 8, 10, 12, 14 [2]

2.3 Classes 0–6, 6–12, 12–24 hold frequencies 9, 10, 12. Which class has the highest density? [2]

Exercises 2.1·2.3

2.1 A histogram bar of width 4 has density 3.5. Find its frequency. [2]

2.1 $4 \times 3.5 = 14$.

2.2 Find the mean of 6, 8, 10, 12, 14 [2]

2.2 $50/5 = 10$.

2.3 Classes 0–6, 6–12, 12–24 hold frequencies 9, 10, 12. Which class has the highest density? [2]

2.3 Densities: $9/6 = 1.5$, $10/6 \approx 1.67$, $12/12 = 1$ – the 6–12 class.

Exercises 2.4·3.1

2.4 $P(\text{rain})$ each independent day is 0.4. Find $P(\text{rain on exactly one of two days})$. [3]

2.5 Of 50 people, 30 speak French, 24 German, 10 both. Find $P(\text{a random person speaks neither})$. [3]

3.1 (*Original.*) A clinic records waiting times 0–10, 10–20, 20–30, 30–60 minutes with frequencies 8, 12, 15, 15. (a) Estimate the mean waiting time. (b) The manager claims “the typical wait is 30–60 minutes because that class holds the most”. Comment. [5]

Exercises 2.4·3.1

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2.4 $0.4 \times 0.6 + 0.6 \times 0.4 = 0.48$.

2.5 Of 50 people, 30 speak French, 24 German, 10 both. Find $P(\text{a random person speaks neither})$. [3]

2.5 $\text{Neither} = 50 - (30 + 24 - 10) = 6$; $P = 6/50 = 3/25$.

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3.1 (a) Midpoints 5, 15, 25, 45: $(40 + 180 + 375 + 675)/50 = 1270/50 = 25.4$ minutes.
 (b) The claim confuses frequency with typicality – 30–60 TIES 20–30 on frequency (15 each), its density is far lower (0.5 vs 1.5), and the estimated mean is 25.4, inside 20–30. The wider class merely collects many patients across a long range.

Exercises 3.2·3.3

3.2 (*Original.*) For the clinic's data: (a) write the cumulative frequencies and plotting points; (b) estimate the median; (c) estimate how many patients waited more than 25 minutes. [6]

3.3 (*Original.*) A game: draw a counter from a bag of 3 gold, 5 silver, replace it, draw again. (a) Find $P(\text{at least one gold})$. (b) The rule changes to WITHOUT replacement: find $P(\text{at least one gold})$ now. (c) Explain why the answer increased. [6]

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3.2 (a) Cumulative frequencies 8, 20, 35, 50 at upper boundaries 10, 20, 30, 60 – the plotted points are (10, 8), (20, 20), (30, 35), (60, 50). (b) The 25th patient lies between 20 (cumulative 20) and 30 (cumulative 35): $20 + 10 \times \frac{5}{15} \approx 23.3$ minutes. (c) At 25 minutes – halfway through the 20–30 class (width 10, frequency 15) – the cumulative frequency is $20 + 15 \times \frac{5}{10} = 27.5$, so about $50 - 27.5 = 22.5$: roughly 23 patients waited more than 25 minutes.

3.3 (*Original.*) A game: draw a counter from a bag of 3 gold, 5 silver, replace it, draw again. (a) Find P(at least one gold). (b) The rule changes to WITHOUT replacement: find P(at least one gold) now. (c) Explain why the answer increased. [6]

3.3 (a) With replacement: $P(\text{no gold}) = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$, so $P(\text{at least one gold}) = \frac{39}{64}$. (b) Without: $P(\text{no gold}) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$, so $P(\text{at least one gold}) = 1 - \frac{5}{14} = \frac{9}{14}$. (c) After a non-gold counter is removed, 3 of the remaining 7 are gold – a better chance ($\frac{3}{7}$) than the $\frac{3}{8}$ with replacement – so the no-gold path got harder ($\frac{5}{14} < \frac{25}{64}$) and drawing at least one gold became likelier. Local review guidance: the correct strand tool chosen (density, midpoints, ogive read, complement); exact values with units; comment questions answered with a number, not an opinion. This is authored classroom guidance.