

Exercise walk-through

5.5

Mixed practice: vectors and transformations

Pearson Edexcel · International GCSE · Mathematics A

1. Topic 5 at a glance

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Foundation 5.2 is the four transformations with complete descriptions; 5.1 (vectors) is Higher-only, from notation through resultants and moduli to geometrical proofs.

Must-know: column $\begin{pmatrix} x \\ y \end{pmatrix}$; modulus $\sqrt{x^2 + y^2}$; route rule $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$; reflection, rotation (centre, angle, direction), translation $\begin{pmatrix} a \\ b \end{pmatrix}$, enlargement (centre, factor).

2. Vectors and resultants

Vectors and resultants

A vector has magnitude and direction; write it $\mathbf{a}$, $\overrightarrow{OA}$ or $\begin{pmatrix} x \\ y \end{pmatrix}$. Scalars stretch and reverse: $2\mathbf{a}$ parallel and twice as long, $-\mathbf{a}$ reversed. Componentwise: $\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$.

Routes add tip-to-tail: with $\overrightarrow{OA} = 3\mathbf{a}$, $\overrightarrow{AB} = 2\mathbf{b}$, $\overrightarrow{BC} = 2\mathbf{c}$ (the specification's example), $\overrightarrow{OC} = 3\mathbf{a} + 2\mathbf{b} + 2\mathbf{c}$ and reversing negates. Modulus by Pythagoras: $|\begin{pmatrix} 3 \\ -5 \end{pmatrix}| = \sqrt{34} \approx 5.83$.

3. Vector proofs

Vector proofs

Write the SAME vector two independent ways and equate. Parallelogram $OABC$ with $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$: $\overrightarrow{OB} = \mathbf{a} + \mathbf{c}$, and the midpoint M of the diagonal gives $\overrightarrow{AM} = \frac{1}{2}(\mathbf{c} - \mathbf{a}) = \frac{1}{2}\overrightarrow{AC}$ - the diagonals bisect each other.

June 2024 2HR Q25 family: vectors $\frac{8}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ and $\frac{9}{4}\mathbf{b} - 2\mathbf{a}$, then $k\overrightarrow{OP}$ equal to a second route: equate the $\mathbf{a}$ and $\mathbf{b}$ coefficients separately and solve - the paper's $k = \frac{5}{8}$. Never assume a midpoint; write λ times and solve.

Exercises 2.1·2.3

2.1 Simplify $3\begin{pmatrix} 2 \\ -1 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ [2]

2.2 Find the modulus of $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$ [2]

2.3 Rotate $(2, -5)$ by 180° about O [1]

Exercises 2.1·2.3

2.1 Simplify $3\binom{2}{-1} - 2\binom{1}{4}$ [2]

2.1 $\binom{6}{-3} - \binom{2}{8} = \binom{4}{-11}$.

2.2 Find the modulus of $\binom{-8}{6}$ [2]

2.2 $\sqrt{64 + 36} = 10$.

2.3 Rotate $(2, -5)$ by 180° about O [1]

2.3 $(-2, 5)$.

Exercises 2.4·3.1

2.4 Enlarge $(-4, 6)$ by scale factor $\frac{1}{2}$ about the origin. [1]

2.5 $\overrightarrow{AB} = 5\mathbf{a} - 2\mathbf{b}$ and $\overrightarrow{BC} = -3\mathbf{a} + 2\mathbf{b}$. Find $\overrightarrow{AC}$ [2]

3.1 (*Original.*) $OAPB$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. (a) Write $\overrightarrow{AB}$ and $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (b) The point G satisfies $\overrightarrow{OG} = \frac{1}{3}\overrightarrow{OP}$. Find $\overrightarrow{AG}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (c) Explain how your answer shows that G does not lie on AB . [5]

Exercises 2.4·3.1

2.4 Enlarge $(-4, 6)$ by scale factor $\frac{1}{2}$ about the origin. [1]

2.4 $(-2, 3)$.

2.5 $\overrightarrow{AB} = 5\mathbf{a} - 2\mathbf{b}$ and $\overrightarrow{BC} = -3\mathbf{a} + 2\mathbf{b}$. Find $\overrightarrow{AC}$ [2]

2.5 $\overrightarrow{AC} = (5\mathbf{a} - 2\mathbf{b}) + (-3\mathbf{a} + 2\mathbf{b}) = 2\mathbf{a}$.

3.1 (*Original.*) $OAPB$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. (a) Write $\overrightarrow{AB}$ and $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (b) The point G satisfies $\overrightarrow{OG} = \frac{1}{3}\overrightarrow{OP}$. Find $\overrightarrow{AG}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (c) Explain how your answer shows that G does not lie on AB . [5]

3.1 (a) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$; $\overrightarrow{OP} = \mathbf{a} + \mathbf{b}$. (b) $\overrightarrow{AG} = \overrightarrow{AO} + \overrightarrow{OG} = -\mathbf{a} + \frac{1}{3}(\mathbf{a} + \mathbf{b}) = \frac{1}{3}\mathbf{b} - \frac{2}{3}\mathbf{a}$.
 (c) If G lay on AB , then $\overrightarrow{AG}$ would be a multiple of $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$; matching coefficients would need the $\mathbf{a}$ and $\mathbf{b}$ coefficients of $\frac{1}{3}\mathbf{b} - \frac{2}{3}\mathbf{a}$ to be equal in size and opposite in sign, but they are $-\frac{2}{3}$ and $\frac{1}{3}$ – not a multiple of $\mathbf{b} - \mathbf{a}$. So G is not on AB (it lies inside the parallelogram, a third of the way along the diagonal).

Exercises 3.2·3.3

3.2 (*Original.*) The point $Q(4, 3)$ is reflected in the y -axis to Q_1 ; Q_1 is then translated by $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ to Q_2 ; Q_2 is then rotated 90° anticlockwise about O to Q_3 . Find Q_3 . [3]

3.3 (*Original, transfer.*) A drone flies 6 km east then 8 km north (vectors from origin). (a) Write the two legs as column vectors. (b) Find the resultant displacement. (c) The drone then flies directly back to the start: write the return leg as a column vector and state its magnitude. [5]

Exercises 3.2-3.3

3.2 (*Original.*) The point $Q(4, 3)$ is reflected in the y -axis to Q_1 ; Q_1 is then translated by $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ to Q_2 ; Q_2 is then rotated 90° anticlockwise about O to Q_3 . Find Q_3 . [3]

3.2 Reflect in the y -axis: $(4, 3) \rightarrow (-4, 3)$. Translate by $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$: $(-4, 3) \rightarrow (-5, 5)$. Rotate 90° anticlockwise about O : $(x, y) \rightarrow (-y, x)$, so $(-5, 5) \rightarrow (-5, -5)$. $Q_3 = (-5, -5)$.

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3.3 (a) $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 8 \end{pmatrix}$. (b) $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$ (km, east 6, north 8). (c) Return leg = $-\begin{pmatrix} 6 \\ 8 \end{pmatrix} = \begin{pmatrix} -6 \\ -8 \end{pmatrix}$; magnitude $\sqrt{36 + 64} = 10$ km.

Local review guidance: each transformation or route step applied in order; exact coordinates and vectors; conclusions state the geometric fact, not just the algebra. This is authored classroom guidance.