

Exercise walk-through

4.6

Three-dimensional trigonometry

Pearson Edexcel · International GCSE · Mathematics A

1. Topic 4 at a glance

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The largest topic: Foundation 4.1-4.11 (angle facts, polygons, measures, constructions, circle language, Pythagoras and 2D trigonometry, mensuration, similarity); Higher adds the circle-theorem programme, obtuse and non-right trigonometry, 3D problems, sectors, cones, spheres and the k -squared/ k -cubed ratios.

Must-know: $a^2 = b^2 + c^2 - 2bc \cos A$; $a/\sin A = b/\sin B$; area $\frac{1}{2}ab \sin C$; arc and sector from $\theta/360$; cone $\frac{1}{3}\pi r^2 h$; sphere $\frac{4}{3}\pi r^3$ and $4\pi r^2$; lengths k , areas k^2 , volumes k^3 .

6. Pythagoras and 2D trigonometry

Pythagoras and 2D trigonometry

$a^2 + b^2 = c^2$; SOH CAH TOA forwards for sides and backwards for angles; elevations rise, depressions fall, alternate angles carry them into the triangle.

Higher: $\sin 130 = \sin 50$, $\cos 130 = -\cos 50$; sine rule for a matching pair; cosine rule for two sides with the included angle or three sides; $\frac{1}{2}ab \sin C$ runs in reverse too - area 15 with $a = 6$, $b = 10$ gives $C = 30$.

Check yourself

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Angle facts with reasons; polygon sums and regular angles; symmetry counts. Bearings and the three compound measures. Constructions with arcs left in; locus regions; scale conversions.

The circle-theorem reason bank applied in chains. Pythagoras and SOH CAH TOA; sine rule, cosine rule and the area formula in both directions. 3D triangles extracted and finished. Sectors, cones, spheres and composite areas. The $k : k^2 : k^3$ ladder both ways.

Exercises 2.1·2.3

2.1 A cuboid has dimensions $3 \text{ cm} \times 4 \text{ cm} \times 12 \text{ cm}$. Find the length of its space diagonal. [3]

2.2 For the cuboid above, find the length of a diagonal of the $3 \text{ cm} \times 4 \text{ cm}$ face. [2]

2.3 In the $6 \times 8 \times 10$ cuboid, find the angle between the space diagonal and the base. [3]

Exercises 2.1·2.3

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$$\mathbf{2.1} \quad \sqrt{9 + 16 + 144} = \sqrt{169} = 13 \text{ cm}.$$

2.2 For the cuboid above, find the length of a diagonal of the $3 \text{ cm} \times 4 \text{ cm}$ face. [2]

$$\mathbf{2.2} \quad \sqrt{9 + 16} = 5 \text{ cm}.$$

2.3 In the $6 \times 8 \times 10$ cuboid, find the angle between the space diagonal and the base. [3]

2.3 The space diagonal meets the base at the end of the base diagonal, so $\tan \theta = 10/10 = 1$ and $\theta = 45^\circ$.

Exercises 2.4·3.2

2.4 A square-based pyramid has base edge 10 cm and all slant edges 13 cm. Find the height of the apex above the centre of the base. [3]

3.1 (*Edexcel 4MA1 1HR June 2024 Q22, adapted.*) The points B , M and P lie in a solid so that $BM = \sqrt{200}$, $BP = \sqrt{244}$ and $MP = 10$ (each length itself found by Pythagoras in a right triangle of the solid). Find $\angle BMP$. [4]

3.2 (*Original.*) A cuboid has base $5 \text{ cm} \times 12 \text{ cm}$ and height 8 cm. G is a bottom corner and B the opposite top corner; C is the base corner diagonally opposite G . (a) Find GC . (b) Find GB . (c) Find the angle between GB and the base diagonal GC . [6]

Exercises 2.4·3.2

2.4 A square-based pyramid has base edge 10 cm and all slant edges 13 cm. Find the height of the apex above the centre of the base. [3]

2.4 Half the base diagonal is $\frac{1}{2}\sqrt{200} = \sqrt{50}$; height = $\sqrt{169 - 50} = \sqrt{119} = 10.9$ cm (3 s.f.).

3.1 (*Edexcel 4MA1 1HR June 2024 Q22, adapted.*) The points B , M and P lie in a solid so that $BM = \sqrt{200}$, $BP = \sqrt{244}$ and $MP = 10$ (each length itself found by Pythagoras in a right triangle of the solid). Find $\angle BMP$. [4]

3.1 Cosine rule at M : $\cos BMP = \frac{200 + 100 - 244}{2\sqrt{200}\sqrt{100}} = \frac{56}{282.8} = 0.198$, so $\angle BMP = 78.6^\circ$.

3.2 (*Original.*) A cuboid has base 5 cm $\times$ 12 cm and height 8 cm. G is a bottom corner and B the opposite top corner; C is the base corner diagonally opposite G . (a) Find GC . (b) Find GB . (c) Find the angle between GB and the base diagonal GC . [6]

3.2 (a) $GC = \sqrt{25 + 144} = 13$ cm. (b) $GB = \sqrt{25 + 144 + 64} = \sqrt{233} = 15.3$ cm (3 s.f.). (c) $\tan \theta = 8/13$, so $\theta = 31.6^\circ$ (1 d.p.).

Exercise 3.3

3.3 (*Original.*) A vertical flagpole of height 12 m stands at the centre O of a horizontal square courtyard of side 16 m. P is a corner of the courtyard. Find the angle of elevation of the top of the flagpole from P . [5]

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3.3 (*Original.*) A vertical flagpole of height 12 m stands at the centre O of a horizontal square courtyard of side 16 m. P is a corner of the courtyard. Find the angle of elevation of the top of the flagpole from P . [5]

3.3 OP is half the base diagonal: $\frac{1}{2} \times 16\sqrt{2} = 11.31$ m. $\tan \theta = 12/11.31 = 1.061$, so $\theta = 46.7^\circ$ (1 d.p.).

Local review guidance: the extracted triangle identified before any computation; surds or 3 s.f. as asked; the angle's vertex named in the notation. This is authored classroom guidance.