

Exercise walk-through

4.5

Pythagoras and trigonometry in 2D

Pearson Edexcel · International GCSE · Mathematics A

1. Topic 4 at a glance

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The largest topic: Foundation 4.1-4.11 (angle facts, polygons, measures, constructions, circle language, Pythagoras and 2D trigonometry, mensuration, similarity); Higher adds the circle-theorem programme, obtuse and non-right trigonometry, 3D problems, sectors, cones, spheres and the k -squared/ k -cubed ratios.

Must-know: $a^2 = b^2 + c^2 - 2bc \cos A$; $a/\sin A = b/\sin B$; area $\frac{1}{2}ab \sin C$; arc and sector from $\theta/360$; cone $\frac{1}{3}\pi r^2 h$; sphere $\frac{4}{3}\pi r^3$ and $4\pi r^2$; lengths k , areas k^2 , volumes k^3 .

6. Pythagoras and 2D trigonometry

Pythagoras and 2D trigonometry

$a^2 + b^2 = c^2$; SOH CAH TOA forwards for sides and backwards for angles; elevations rise, depressions fall, alternate angles carry them into the triangle.

Higher: $\sin 130 = \sin 50$, $\cos 130 = -\cos 50$; sine rule for a matching pair; cosine rule for two sides with the included angle or three sides; $\frac{1}{2}ab \sin C$ runs in reverse too - area 15 with $a = 6$, $b = 10$ gives $C = 30$.

Check yourself

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Angle facts with reasons; polygon sums and regular angles; symmetry counts. Bearings and the three compound measures. Constructions with arcs left in; locus regions; scale conversions.

The circle-theorem reason bank applied in chains. Pythagoras and SOH CAH TOA; sine rule, cosine rule and the area formula in both directions. 3D triangles extracted and finished. Sectors, cones, spheres and composite areas. The $k : k^2 : k^3$ ladder both ways.

Exercises 2.1·2.3

2.1 A right triangle has legs 9 cm and 12 cm. Find the hypotenuse. [2]

2.2 A 6 m ladder leans against a wall at 65° to the ground. How high up the wall does it reach? [2]

2.3 Write down the value of $\sin 130^\circ$ as a ratio of an acute angle. [1]

Exercises 2.1·2.3

2.1 A right triangle has legs 9 cm and 12 cm. Find the hypotenuse. [2]

$$\mathbf{2.1} \quad \sqrt{81 + 144} = \sqrt{225} = 15 \text{ cm.}$$

2.2 A 6 m ladder leans against a wall at 65° to the ground. How high up the wall does it reach? [2]

$$\mathbf{2.2} \quad 6 \sin 65^\circ = 5.44 \text{ m (3 s.f.).}$$

2.3 Write down the value of $\sin 130^\circ$ as a ratio of an acute angle. [1]

$$\mathbf{2.3} \quad \sin 50^\circ.$$

Exercises 2.4·3.1

2.4 In triangle ABC , $b = 5$, $c = 7$ and $\angle A = 60^\circ$. Find a . [3]

2.5 Triangle area is 15 cm^2 with $a = 6 \text{ cm}$ and $b = 10 \text{ cm}$. Find the included angle C . [3]

3.1 (*Original.*) A tower is 50 m tall. From a point 80 m from its base on level ground, find the angle of elevation of the top of the tower. [3]

Exercises 2.4·3.1

2.4 In triangle ABC , $b = 5$, $c = 7$ and $\angle A = 60^\circ$. Find a . [3]

2.4 $a^2 = 25 + 49 - 70 \cos 60^\circ = 74 - 35 = 39$, so $a = 6.24$ (3 s.f.).

2.5 Triangle area is 15 cm^2 with $a = 6 \text{ cm}$ and $b = 10 \text{ cm}$. Find the included angle C . [3]

2.5 $\frac{1}{2} \times 6 \times 10 \times \sin C = 15$ gives $\sin C = 0.5$, so $C = 30^\circ$.

3.1 (*Original.*) A tower is 50 m tall. From a point 80 m from its base on level ground, find the angle of elevation of the top of the tower. [3]

3.1 $\tan \theta = 50/80 = 0.625$, so $\theta = 32.0^\circ$ (1 d.p.).

Exercises 3.2·3.3

3.2 (*Original.*) In triangle PQR : $PQ = 8$ cm, $\angle P = 40^\circ$ and $\angle Q = 60^\circ$. (a) Find $\angle R$.
(b) Use the sine rule to find QR . (c) Find the area of PQR . [6]

3.3 (*Original.*) A ship sails 30 km on bearing 040° from port A to a lighthouse B , then 45 km on bearing 130° to port C . (a) Show that $\angle ABC = 90^\circ$. (b) Find AC . [5]

Exercises 3.2·3.3

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3.2 (a) $\angle R = 180 - 40 - 60 = 80^\circ$. (b) QR is opposite $\angle P$: $QR/\sin 40 = 8/\sin 80$, so $QR = 8 \sin 40/\sin 80 = 5.22$ cm (3 s.f.). (c) Using sides $PQ = 8$ and $QR = 5.22$ with included angle $Q = 60^\circ$: $\text{area} = \frac{1}{2} \times 8 \times 5.22 \times \sin 60 = 18.1$ cm² (3 s.f.).

3.3 (*Original.*) A ship sails 30 km on bearing 040° from port A to a lighthouse B , then 45 km on bearing 130° to port C . (a) Show that $\angle ABC = 90^\circ$. (b) Find AC . [5]

3.3 (a) At B , the back-bearing of A is $040 + 180 = 220^\circ$; C bears 130° , so $\angle ABC = 220 - 130 = 90^\circ$. (b) $AC = \sqrt{30^2 + 45^2} = \sqrt{2925} = 54.1$ km (3 s.f.).

Local review guidance: the correct rule chosen for the given parts; 3 s.f. accuracy with units; bearings angles found via back-bearings, not guesses. This is authored classroom guidance.