

Exercise walk-through

3.6

Mixed Higher practice: sequences, functions and graphs

Pearson Edexcel · International GCSE · Mathematics A

1. Topic 3 at a glance

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Foundation 3.1 and 3.3 are shared; Higher adds function notation (3.2), the full graphs programme (3.3) and calculus (3.4). On the June 2024 Higher papers this strand is about a third of each paper.

Must-know: $u_n = a + (n - 1)d$ and $S_n = \frac{n}{2}[2a + (n - 1)d]$; $fg(x) = f(g(x))$ with g first; perpendicular gradients multiply to -1 ; intersections solve $y_2 - y_1 = 0$;
 $\frac{d}{dx}(ax^n) = anx^{n-1}$.

7. Curve families and the four transformations

Curve families and the four transformations

Recognise cubics $y = Ax^3 + Bx^2 + Cx + D$, reciprocals A/x , A/x^2 , A/x^3 , the sine, cosine and tangent graphs (degrees, period 360 or 180), and the origin-centred circle $x^2 + y^2 = r^2$.

$y = f(x) + a$ translates up; $y = f(x + a)$ translates left - inside acts on x the other way; $y = af(x)$ stretches vertically; $y = f(ax)$ compresses horizontally. For $f(x) = x^2$, $f(2x) = 4x^2$ is narrower, not taller.

Check yourself

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Generate and rule sequences; recover a and d ; sum with $S_n = \frac{n}{2}[2a + (n-1)d]$ including algebraic terms. State domains with exclusions; build fg with g first; invert one-to-one functions with the inverse's domain.

Midpoints, two-point gradients, parallel and perpendicular equations. Cubics, reciprocals, trig graphs and the circle; the four transformations read both ways; intersections via $y_2 - y_1 = 0$. Differentiate integer powers; find and classify stationary points; apply $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt}$ to motion.

Exercises 2.1·2.3

2.1 Find a rule for the n th term of 6, 11, 16, 21, ... [2]

2.2 Find the sum of the first 12 terms of $-3 + 1 + 5 + 9 + \dots$ [3]

2.3 Find the equation of the line perpendicular to $y = \frac{1}{4}x - 7$ through the point (5, 2). [3]

Exercises 2.1·2.3

2.1 Find a rule for the n th term of 6, 11, 16, 21, ... [2]

2.1 Differences are 5: $u_n = 6 + 5(n - 1) = 5n + 1$.

2.2 Find the sum of the first 12 terms of $-3 + 1 + 5 + 9 + \dots$ [3]

2.2 $a = -3$, $d = 4$: $S_{12} = 6[-6 + 11 \times 4] = 6 \times 38 = 228$.

2.3 Find the equation of the line perpendicular to $y = \frac{1}{4}x - 7$ through the point (5, 2). [3]

2.3 Perpendicular gradient -4 : $y - 2 = -4(x - 5)$, so $y = -4x + 22$.

Exercises 2.4·3.1

2.4 For $f(x) = \sqrt{3x - 5}$, write down the smallest value of x allowed in the domain. [2]

2.5 For $y = \frac{1}{4}x^4 - 2x^2$, find the x coordinates of all stationary points. [3]

3.1 (*Original.*) $f(x) = x^2 - 8x + 20$. (a) Write down the coordinates of the minimum point of $y = f(x)$. (b) The function g is defined by $g(x) = 2f(x - 1)$. Find the coordinates of the minimum point of $y = g(x)$. [5]

Exercises 2.4·3.1

2.4 For $f(x) = \sqrt{3x-5}$, write down the smallest value of x allowed in the domain. [2]

2.4 $3x - 5 \geq 0$, so $x \geq \frac{5}{3}$.

2.5 For $y = \frac{1}{4}x^4 - 2x^2$, find the x coordinates of all stationary points. [3]

2.5 $\frac{dy}{dx} = x^3 - 4x = x(x-2)(x+2) = 0$: $x = -2, 0, 2$.

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3.1 (a) $f(x) = (x-4)^2 + 4$: minimum $(4, 4)$. (b) $f(x-1)$ shifts right by 1 to $(5, 4)$; multiplying by 2 stretches vertically to $(5, 8)$.

Exercises 3.2·3.3

3.2 (*Original.*) The line $y = 2x - 3$ meets the curve $y = x^2 - x - 6$. (a) Show that the x coordinates of the intersection points satisfy $x^2 - 3x - 3 = 0$. (b) Find the coordinates of both points, giving exact answers. [5]

3.3 (*Original.*) A theatre has 18 seats in row 1, and each row after it has 4 more seats than the row in front. (a) Find the number of seats in row 25. (b) Find the total number of seats in the first 25 rows. [5]

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3.2 (a) $x^2 - x - 6 = 2x - 3$ gives $x^2 - 3x - 3 = 0$. (b) $x = \frac{3 \pm \sqrt{9+12}}{2} = \frac{3 \pm \sqrt{21}}{2}$. With $y = 2x - 3 = \pm\sqrt{21}$: $\left(\frac{3+\sqrt{21}}{2}, \sqrt{21}\right)$ and $\left(\frac{3-\sqrt{21}}{2}, -\sqrt{21}\right)$.

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3.3 (a) $18 + 24 \times 4 = 114$ seats. (b) $S_{25} = \frac{25}{2} [36 + 24 \times 4] = \frac{25}{2} \times 132 = 1650$.

Exercise 3.4

3.4 (*Original.*) A particle moves so that its displacement after t seconds is $s = 2t^3 - 15t^2 + 24t$ metres. (a) Find an expression for v in terms of t . (b) Find the values of t at which the particle is instantaneously at rest. [4]

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3.4 (a) $v = 6t^2 - 30t + 24$. (b) $6(t - 1)(t - 4) = 0$: $t = 1$ and $t = 4$ seconds.

Local review guidance: the strand's set-up equation before arithmetic; exact or correctly rounded values with units in context; combined transformations applied in the right order – shift then stretch. This is authored classroom guidance.