

Exercise walk-through

3.5

Graphs and transformations

Pearson Edexcel · International GCSE · Mathematics A

1. Topic 3 at a glance

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Foundation 3.1 and 3.3 are shared; Higher adds function notation (3.2), the full graphs programme (3.3) and calculus (3.4). On the June 2024 Higher papers this strand is about a third of each paper.

Must-know: $u_n = a + (n - 1)d$ and $S_n = \frac{n}{2}[2a + (n - 1)d]$; $fg(x) = f(g(x))$ with g first; perpendicular gradients multiply to -1 ; intersections solve $y_2 - y_1 = 0$;
 $\frac{d}{dx}(ax^n) = anx^{n-1}$.

7. Curve families and the four transformations

Curve families and the four transformations

Recognise cubics $y = Ax^3 + Bx^2 + Cx + D$, reciprocals A/x , A/x^2 , A/x^3 , the sine, cosine and tangent graphs (degrees, period 360 or 180), and the origin-centred circle $x^2 + y^2 = r^2$.

$y = f(x) + a$ translates up; $y = f(x + a)$ translates left - inside acts on x the other way; $y = af(x)$ stretches vertically; $y = f(ax)$ compresses horizontally. For $f(x) = x^2$, $f(2x) = 4x^2$ is narrower, not taller.

Check yourself

Check yourself

Generate and rule sequences; recover a and d ; sum with $S_n = \frac{n}{2}[2a + (n-1)d]$ including algebraic terms. State domains with exclusions; build fg with g first; invert one-to-one functions with the inverse's domain.

Midpoints, two-point gradients, parallel and perpendicular equations. Cubics, reciprocals, trig graphs and the circle; the four transformations read both ways; intersections via $y_2 - y_1 = 0$. Differentiate integer powers; find and classify stationary points; apply $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt}$ to motion.

Exercises 2.1·2.3

2.1 The curve $y = \frac{6}{x}$ meets the line $x = 2$. Find the coordinates of the point of intersection. [2]

2.2 For $f(x) = x^2$, write down the equation of $y = f(x - 3) + 1$ and the coordinates of its minimum point. [3]

2.3 For $f(x) = x^2 + 4x$, write down the equation of $y = f(2x)$. [2]

Exercises 2.1·2.3

2.1 The curve $y = \frac{6}{x}$ meets the line $x = 2$. Find the coordinates of the point of intersection. [2]

2.1 $y = 6/2 = 3$: $(2, 3)$.

2.2 For $f(x) = x^2$, write down the equation of $y = f(x - 3) + 1$ and the coordinates of its minimum point. [3]

2.2 $y = (x - 3)^2 + 1$; minimum $(3, 1)$.

2.3 For $f(x) = x^2 + 4x$, write down the equation of $y = f(2x)$. [2]

2.3 $f(2x) = (2x)^2 + 4(2x) = 4x^2 + 8x$.

Exercises 2.4·3.1

2.4 Describe fully the transformation that maps $y = \sin x$ onto $y = \sin(x - 30^\circ)$. [2]

2.5 Write down the quadratic equation whose solutions are the x coordinates of the intersections of $y = 2x + 1$ and $y = x^2 + 3x - 2$. [2]

3.1 (*Edexcel 4MA1 2HR June 2024 Q22, adapted.*) Find the coordinates of the points where the line $y = 3x - 1$ meets the curve $x^2 + y^2 = y + 11$. [5]

Exercises 2.4·3.1

2.4 Describe fully the transformation that maps $y = \sin x$ onto $y = \sin(x - 30^\circ)$. [2]

2.4 A translation of 30° to the right (horizontal shift along x); shape, period and amplitude unchanged.

2.5 Write down the quadratic equation whose solutions are the x coordinates of the intersections of $y = 2x + 1$ and $y = x^2 + 3x - 2$. [2]

2.5 $x^2 + 3x - 2 = 2x + 1$, so $x^2 + x - 3 = 0$.

3.1 (*Edexcel 4MA1 2HR June 2024 Q22, adapted.*) Find the coordinates of the points where the line $y = 3x - 1$ meets the curve $x^2 + y^2 = y + 11$. [5]

3.1 Substitute $y = 3x - 1$: $x^2 + (3x - 1)^2 = (3x - 1) + 11$, so $10x^2 - 6x + 1 = 3x + 10$ and $10x^2 - 9x - 9 = 0$. Factorising, $(5x + 3)(2x - 3) = 0$, so $x = -\frac{3}{5}$ or $x = \frac{3}{2}$. With $y = 3x - 1$: $(-\frac{3}{5}, -\frac{14}{5})$ and $(\frac{3}{2}, \frac{7}{2})$.

Exercises 3.2·3.3

3.2 (*Original.*) $f(x) = x^2 - 6x + 11$. (a) Write down the coordinates of the minimum point of $y = f(x)$. (b) The graph of $y = f(x + k)$ has its minimum point at $(0, 2)$: find k . (c) Write down the coordinates of the minimum point of $y = f(2x)$. [6]

3.3 (*Original.*) The line $y = x + 2$ meets the curve $y = x^3 - 4x$. (a) Show that $x = -2$ is one solution of $x^3 - 5x - 2 = 0$. (b) Find the coordinates of all three points of intersection. [5]

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3.2 (a) $f(x) = (x - 3)^2 + 2$, so the minimum is $(3, 2)$. (b) $f(x + k)$ shifts the graph k units left, so the minimum sits at $3 - k = 0$: $k = 3$. (c) $f(2x)$ halves the x -values: minimum $(1.5, 2)$.

3.3 (*Original.*) The line $y = x + 2$ meets the curve $y = x^3 - 4x$. (a) Show that $x = -2$ is one solution of $x^3 - 5x - 2 = 0$. (b) Find the coordinates of all three points of intersection. [5]

3.3 (a) $(-2)^3 - 5(-2) - 2 = -8 + 10 - 2 = 0$. ✓ (b) $x^3 - 5x - 2 = (x + 2)(x^2 - 2x - 1)$, so $x = 1 \pm \sqrt{2}$ as well. Points: $(-2, 0)$, $(1 + \sqrt{2}, 3 + \sqrt{2})$ and $(1 - \sqrt{2}, 3 - \sqrt{2})$.

Local review guidance: substitution of the linear into the non-linear before any expansion; exact coordinates including both points; transformation descriptions name the direction and the variable acted on. This is authored classroom guidance.