

Exercise walk-through

2.6

Mixed Higher practice: equations, formulae and identities

Pearson Edexcel · International GCSE · Mathematics A

1. Topic 2 at a glance

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Foundation rows 2.1-2.4 and 2.6-2.8 are shared; proportion (2.5) is Higher-only, and Higher extends to fractional indices, algebraic fractions, completing the square, proofs, linear-quadratic pairs and quadratic inequalities.

Must-know: index laws on one base; change the subject by balanced operations (powers need rooting); factor before cancelling; $a(x + b)^2 + c$ exposes the vertex; critical values then the parabola's sign.

8. Solving quadratics

Solving quadratics

Three routes: factorise ($x^2 + x - 30 = (x + 6)(x - 5)$); the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ when factorising fails ($2x^2 + 6x - 1 = 0$ gives $x = \frac{-3 \pm \sqrt{11}}{2}$); completing the square solves and gives the vertex at once.

Form equations from context - the lawn $w(w + 3) = 70$ gives $w^2 + 3w - 70 = (w + 10)(w - 7)$, so $w = 7$. Keep exact answers; reject roots the context forbids.

Check yourself

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Index laws on one base including fractional and negative powers; substitution, expansion (double and triple brackets) and factorising. Rearrange formulae including powers and subject-twice; set up and solve proportion models.

Exact simultaneous pairs by elimination; algebraic fractions factor-cancel-invert; recurring-decimal and identity proofs; complete the square and read vertex-form optima; solve quadratic inequalities with number-line sets; linear-quadratic pairs keeping both points.

Exercises 2.1·2.3

2.1 Simplify $(x^3)^2 \div x^{-4}$ [2]

2.2 Make g the subject of $T = 2\pi\sqrt{\frac{l}{g}}$ [3]

2.3 Simplify $\frac{x^2 + 2x - 8}{x^2 - 16}$ [3]

Exercises 2.1·2.3

2.1 Simplify $(x^3)^2 \div x^{-4}$ [2]

2.1 $x^6 \div x^{-4} = x^{6+4} = x^{10}.$

2.2 Make g the subject of $T = 2\pi\sqrt{\frac{l}{g}}$ [3]

2.2 $T^2 = \frac{4\pi^2 l}{g}$, so $gT^2 = 4\pi^2 l$ and $g = \frac{4\pi^2 l}{T^2}.$

2.3 Simplify $\frac{x^2 + 2x - 8}{x^2 - 16}$ [3]

2.3 $\frac{(x+4)(x-2)}{(x-4)(x+4)} = \frac{x-2}{x-4}$ (for $x \neq \pm 4$).

Exercises 2.4·3.1

2.4 Write $3x^2 - 12x + 5$ in the form $a(x + b)^2 + c$ [3]

2.5 Solve $4x^2 - 25 \leq 0$ [3]

3.1 (*Original.*) Given that $\frac{2^{3n} \times 2^{n+4}}{2^{2n+1}} = 32$, find the value of n . [3]

Exercises 2.4·3.1

2.4 Write $3x^2 - 12x + 5$ in the form $a(x + b)^2 + c$ [3]

2.4 $3(x^2 - 4x) + 5 = 3(x - 2)^2 - 12 + 5 = 3(x - 2)^2 - 7.$

2.5 Solve $4x^2 - 25 \leq 0$ [3]

2.5 $(2x - 5)(2x + 5) \leq 0$: critical values $\pm\frac{5}{2}$; the parabola opens upward, so $-\frac{5}{2} \leq x \leq \frac{5}{2}.$

3.1 (*Original.*) Given that $\frac{2^{3n} \times 2^{n+4}}{2^{2n+1}} = 32$, find the value of n . [3]

3.1 Right side: $32 = 2^5$. Indices: $3n + (n + 4) - (2n + 1) = 2n + 3 = 5$, so $n = 1$.

Exercises 3.2·3.4

3.2 (*Original.*) Write $\frac{x+1}{x-2} - \frac{3}{x+1}$ as a single fraction in its simplest form. [3]

3.3 (*Original.*) A ball is thrown so that its height after t seconds is $h = 20t - 5t^2$ metres.
(a) Write h in the form $a(t+b)^2 + c$. (b) State the maximum height and when it occurs.[4]

3.4 (*Original.*) The surface area of a closed cylinder is given by $A = 2\pi r^2 + 2\pi rh$. Make h the subject, then find h when $A = 120\pi$ and $r = 3$. [4]

Exercises 3.2·3.4

3.2 (*Original.*) Write $\frac{x+1}{x-2} - \frac{3}{x+1}$ as a single fraction in its simplest form. [3]

$$\mathbf{3.2} \quad \frac{(x+1)^2 - 3(x-2)}{(x-2)(x+1)} = \frac{x^2 + 2x + 1 - 3x + 6}{(x-2)(x+1)} = \frac{x^2 - x + 7}{(x-2)(x+1)} \text{ (already simplest).}$$

3.3 (*Original.*) A ball is thrown so that its height after t seconds is $h = 20t - 5t^2$ metres.

(a) Write h in the form $a(t+b)^2 + c$. (b) State the maximum height and when it occurs. [4]

3.3 (a) $h = -5(t^2 - 4t) = -5(t-2)^2 + 20$. (b) Maximum height 20 m at $t = 2$ seconds.

3.4 (*Original.*) The surface area of a closed cylinder is given by $A = 2\pi r^2 + 2\pi rh$. Make h the subject, then find h when $A = 120\pi$ and $r = 3$. [4]

$$\mathbf{3.4} \quad h = \frac{A - 2\pi r^2}{2\pi r}; \text{ with } A = 120\pi, r = 3: h = \frac{120\pi - 18\pi}{6\pi} = \frac{102\pi}{6\pi} = 17 \text{ metres.}$$

Local review guidance: the strand's set-up step before arithmetic; exact simplified forms with stated exclusions; optimum answers read from vertex form, with units in context. This is authored classroom guidance.