

Exercise walk-through

2.4

Completing the square and solving quadratics

Pearson Edexcel · International GCSE · Mathematics A

1. Topic 2 at a glance

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Foundation rows 2.1-2.4 and 2.6-2.8 are shared; proportion (2.5) is Higher-only, and Higher extends to fractional indices, algebraic fractions, completing the square, proofs, linear-quadratic pairs and quadratic inequalities.

Must-know: index laws on one base; change the subject by balanced operations (powers need rooting); factor before cancelling; $a(x + b)^2 + c$ exposes the vertex; critical values then the parabola's sign.

8. Solving quadratics

Solving quadratics

Three routes: factorise ($x^2 + x - 30 = (x + 6)(x - 5)$); the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ when factorising fails ($2x^2 + 6x - 1 = 0$ gives $x = \frac{-3 \pm \sqrt{11}}{2}$); completing the square solves and gives the vertex at once.

Form equations from context - the lawn $w(w + 3) = 70$ gives $w^2 + 3w - 70 = (w + 10)(w - 7)$, so $w = 7$. Keep exact answers; reject roots the context forbids.

Check yourself

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Index laws on one base including fractional and negative powers; substitution, expansion (double and triple brackets) and factorising. Rearrange formulae including powers and subject-twice; set up and solve proportion models.

Exact simultaneous pairs by elimination; algebraic fractions factor-cancel-invert; recurring-decimal and identity proofs; complete the square and read vertex-form optima; solve quadratic inequalities with number-line sets; linear-quadratic pairs keeping both points.

Exercises 2.1·2.3

2.1 Write $x^2 + 6x + 5$ in the form $(x + b)^2 + c$ [2]

2.2 Hence state the coordinates of the minimum point of $y = x^2 + 6x + 5$ [1]

2.3 Solve $x^2 + x - 30 = 0$ by factorisation [2]

Exercises 2.1·2.3

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2.1 $x^2 + 6x + 5 = (x + 3)^2 - 9 + 5 = (x + 3)^2 - 4.$

2.2 Hence state the coordinates of the minimum point of $y = x^2 + 6x + 5$ [1]

2.2 Minimum $(-3, -4).$

2.3 Solve $x^2 + x - 30 = 0$ by factorisation [2]

2.3 $(x + 6)(x - 5) = 0$: $x = -6$ or $5.$

Exercises 2.4·3.1

2.4 Solve $2x^2 + 6x - 1 = 0$, giving exact answers [3]

2.5 Write $2x^2 - 11x + 9$ in the form $a(x + b)^2 + c$ [3]

3.1 (*Edexcel 4MA1 1HR June 2024 Q20, adapted.*) (a) Write $2x^2 - 11x + 9$ in the form $a(x + b)^2 + c$. (b) The graph of $y = f(x - 3)$, where $f(x) = 2x^2 - 11x + 9$, has a minimum point. Find the coordinates of this minimum point. [5]

Exercises 2.4·3.1

2.4 Solve $2x^2 + 6x - 1 = 0$, giving exact answers [3]

$$\mathbf{2.4} \quad x = \frac{-6 \pm \sqrt{36 + 8}}{4} = \frac{-6 \pm \sqrt{44}}{4} = \frac{-3 \pm \sqrt{11}}{2}.$$

2.5 Write $2x^2 - 11x + 9$ in the form $a(x + b)^2 + c$ [3]

$$\mathbf{2.5} \quad 2x^2 - 11x + 9 = 2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + 9 = 2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}.$$

3.1 (*Edexcel 4MA1 1HR June 2024 Q20, adapted.*) (a) Write $2x^2 - 11x + 9$ in the form $a(x + b)^2 + c$. (b) The graph of $y = f(x - 3)$, where $f(x) = 2x^2 - 11x + 9$, has a minimum point. Find the coordinates of this minimum point. [5]

3.1 (a) As 2.5: $2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$. (b) f has its minimum at $\left(\frac{11}{4}, -\frac{49}{8}\right)$; $f(x - 3)$ shifts 3 right, so the minimum is $\left(\frac{23}{4}, -\frac{49}{8}\right) = (5.75, -6.125)$.

Exercises 3.2·3.3

3.2 (*Original.*) A rectangular lawn has length 3 m more than its width. Its area is 70 m^2 . Form and solve a quadratic equation to find the width and length. [4]

3.3 (*Original.*) Solve $3x^2 = 4x + 2$ giving answers correct to 2 decimal places. [3]

Exercises 3.2·3.3

3.2 (*Original.*) A rectangular lawn has length 3 m more than its width. Its area is 70 m^2 . Form and solve a quadratic equation to find the width and length. [4]

3.2 Width w : $w(w + 3) = 70$, so $w^2 + 3w - 70 = 0 = (w + 10)(w - 7)$. Width 7 m (positive root), length 10 m. Check: $7 \times 10 = 70$. ✓

3.3 (*Original.*) Solve $3x^2 = 4x + 2$ giving answers correct to 2 decimal places. [3]

$$\mathbf{3.3} \quad 3x^2 - 4x - 2 = 0: \quad x = \frac{4 \pm \sqrt{16 + 24}}{6} = \frac{4 \pm \sqrt{40}}{6} = 1.72 \text{ or } -0.39 \text{ (2 d.p.)}.$$

Local review guidance: half-the-coefficient structure with the correcting constant; exact values unless decimals are asked; context rejects the impossible root with a reason. This is authored classroom guidance.