

Vectors and transformation geometry

Pearson Edexcel · International GCSE · Mathematics A

4MA1 topic 5 (specification Issue 2, November 2017). Foundation 5.2 is the four transformations with full descriptions; 5.1 (vectors) is Higher-only and runs from notation to resultants, magnitudes and simple geometrical proofs. On the June 2024 Higher paper the vector question (2HR Q25) is the classic A/A* finisher: a chain of points expressed in **a** and **b**, then an unknown ratio k found by writing the same vector two independent ways.

Must-know for topic 5:

Family	Formulae
column vector	$\begin{pmatrix} x \\ y \end{pmatrix}$; scalar multiple $k\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$
magnitude	$\left \begin{pmatrix} x \\ y \end{pmatrix} \right = \sqrt{x^2 + y^2}$
route rule	$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$; reversing: $\overrightarrow{BA} = -\overrightarrow{AB}$
transformations	reflection in $x = k$, $y = k$, $y = x$; rotation (centre, angle, direction); translation $\begin{pmatrix} a \\ b \end{pmatrix}$; enlargement (centre, factor k)

Vectors: notation to resultants

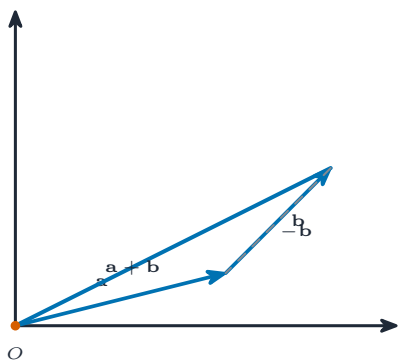
A **vector** has both magnitude and direction (a scalar has only size). Write it bold **a**, underlined, or as $\overrightarrow{OA}$; as a **column vector** $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ – across, then up.

Scalar multiples stretch: $2\mathbf{a}$ is twice as long and parallel; $-\mathbf{a}$ reverses direction; $\frac{1}{2}\mathbf{a}$ halves it.

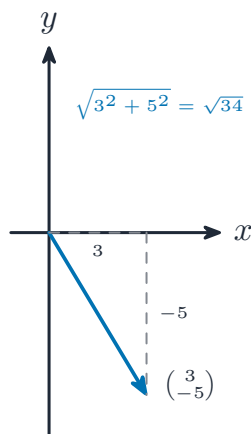
Addition is tip-to-tail: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ – the route matters, not the stops. Reverse a vector by negating: $\overrightarrow{BA} = -\overrightarrow{AB}$. The **resultant** of several legs is the single start-to-end vector: with $\overrightarrow{OA} = 3\mathbf{a}$, $\overrightarrow{AB} = 2\mathbf{b}$ and $\overrightarrow{BC} = 2\mathbf{c}$ (the specification's own example),

$$\overrightarrow{OC} = 3\mathbf{a} + 2\mathbf{b} + 2\mathbf{c}, \quad \overrightarrow{CA} = -\overrightarrow{OC} + \overrightarrow{OA} \text{ summed along the route.}$$

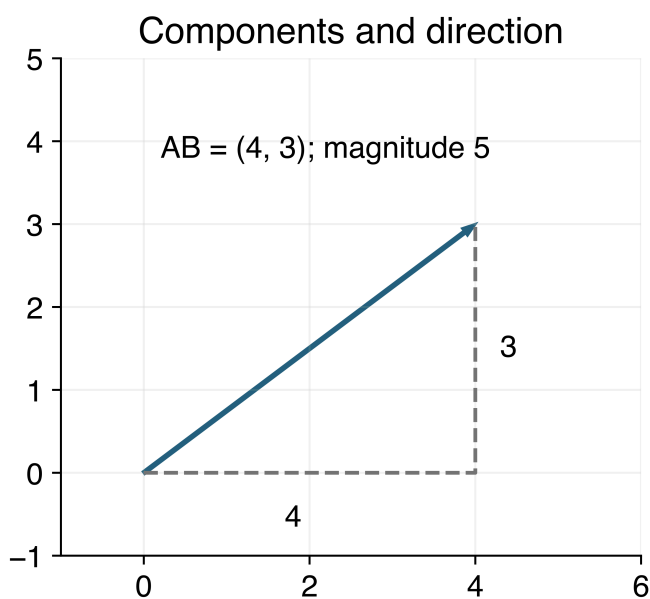
The **modulus** (magnitude) of $\begin{pmatrix} x \\ y \end{pmatrix}$ is its length by Pythagoras: $\sqrt{x^2 + y^2}$, so $\left| \begin{pmatrix} 3 \\ -5 \end{pmatrix} \right| = \sqrt{34} \approx 5.83$. Column-vector arithmetic is componentwise: $\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$.



tip-to-tail: the route adds, the stops do not



modulus = length, by Pythagoras



Original native-lesson illustration; labels belong to its worked example.

Vector geometry proofs

Vector proofs show parallel or half-length relations by writing the SAME vector two ways. **Worked example (original).** $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$; M is the midpoint of diagonal OB . Prove that the diagonals bisect each other.

Diagonal: $\overrightarrow{OB} = \mathbf{a} + \mathbf{c}$ (route $O \rightarrow A \rightarrow B$, since $\overrightarrow{AB} = \mathbf{c}$ in a parallelogram). So $\overrightarrow{OM} = \frac{1}{2}(\mathbf{a} + \mathbf{c})$. From A : $\overrightarrow{AM} = \overrightarrow{AO} + \overrightarrow{OM} = -\mathbf{a} + \frac{1}{2}(\mathbf{a} + \mathbf{c}) = \frac{1}{2}(\mathbf{c} - \mathbf{a})$. But the other diagonal has $\overrightarrow{AC} = -\mathbf{a} + \mathbf{c} = \mathbf{c} - \mathbf{a}$, so $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AC}$: M is the midpoint of AC too. Both diagonals share the midpoint – bisected. ✓

Worked example (2HR June 2024 Q25 family, adapted). Points on a line built from $\mathbf{a}$ and $\mathbf{b}$ typically ask for two things: a vector such as $\frac{8}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ or $\frac{9}{4}\mathbf{b} - 2\mathbf{a}$ in simplest form, then an unknown k with Y on a line: write $\overrightarrow{OY}$ two independent ways – for the paper, $k(\frac{8}{5}\mathbf{a} + \frac{3}{5}\mathbf{b})$ and a route through the other points – equate coefficients of $\mathbf{a}$ (and $\mathbf{b}$), and solve: $k = \frac{5}{8}$. Never assume a midpoint: let it be λ times the leg and EQUATE.

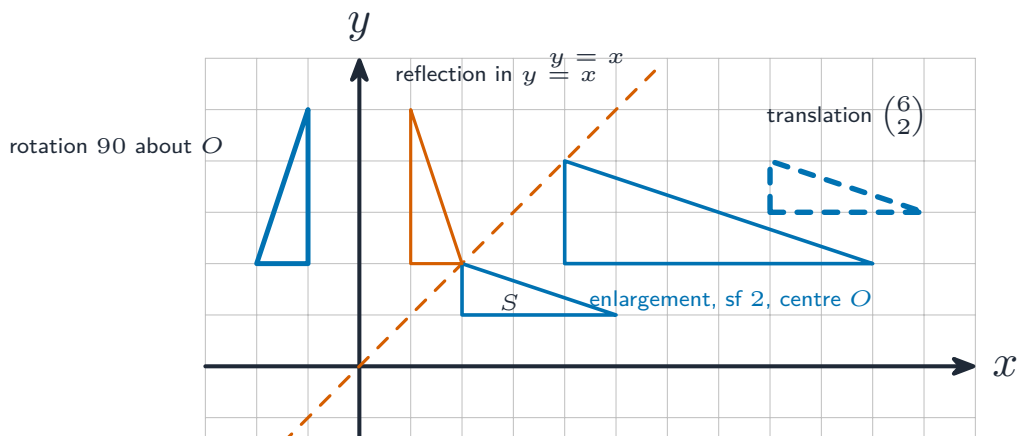
Transformations: rotation, reflection, translation

A **rotation** needs centre, angle and direction (anticlockwise positive, clockwise negative); a **reflection** needs a **mirror line** ($x = 1$, $y = 2$, $y = x$, $y - x = 0$); a **translation** needs a distance and direction, written as the column vector $\begin{pmatrix} a \\ b \end{pmatrix}$. Rotations, reflections and translations preserve lengths and angles, so the image is **congruent** to the original.

Coordinate rules for the quick cases: reflect in the x -axis: $(x, y) \rightarrow (x, -y)$; in the y -axis: $(x, y) \rightarrow (-x, y)$; in $y = x$: $(x, y) \rightarrow (y, x)$; rotate 90° anticlockwise about O : $(x, y) \rightarrow (-y, x)$; rotate 180° : $(x, y) \rightarrow (-x, -y)$; translate by $\begin{pmatrix} a \\ b \end{pmatrix}$: $(x, y) \rightarrow (x + a, y + b)$. Tracing paper does the rest – but the DESCRIPTION must name every required element.

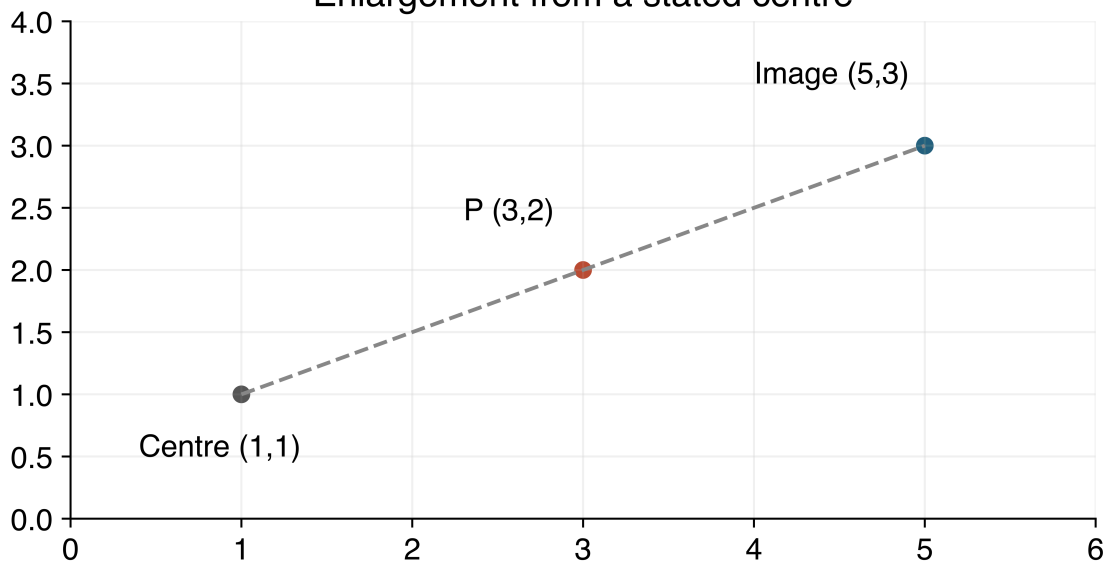
Enlargement and describing transformations

An **enlargement** is specified by a **centre of enlargement** and a positive **scale factor** (fractions included, with or without the centre marked): $(x, y) \rightarrow (kx, ky)$ about the origin. Enlargements preserve angles but NOT lengths – the image is similar, not congruent. A **complete description** names the transformation and every parameter: "rotation, 90° anticlockwise about $(0, 1)$ "; "reflection in the line $y = x$ "; "translation by $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ "; "enlargement, scale factor 2, centre $(1, 1)$ ". Missing the centre or direction loses the mark.



a complete description names the transformation AND every parameter

Enlargement from a stated centre



Original native-lesson illustration; labels belong to its worked example.

Checklist

- Use vector notation, column vectors, scalar multiples and componentwise addition/subtraction (H 5.1 A–D).
- Find resultants along a route; reverse by negating (H 5.1 F).
- Compute moduli by Pythagoras (H 5.1 E; spec example $(3, -5) \rightarrow \sqrt{34}$).
- Prove midpoint, parallel and ratio facts by writing one vector two independent ways – never assume a midpoint (H 5.1 G; June 2024 2HR Q25).
- Rotate, reflect and translate, using column vectors for translations; state congruence preservation (F 5.2 A–I).
- Enlarge with centre and scale factor (fractions included); describe any transformation completely (F 5.2 J–M).

Boundary notes: vectors at this level are in two dimensions with positive scale factors only; negative enlargement factors belong to later study. Foundation candidates meet 5.2 only. Worked examples are adapted from the reviewed June 2024 Higher paper or original (see the meta’s assessment evidence); sheet questions are original unless labelled.