

Geometry and trigonometry

Pearson Edexcel · International GCSE · Mathematics A

4MA1 topic 4 (specification Issue 2, November 2017) is the largest topic: Foundation rows 4.1–4.11 cover angle facts, polygons, measures, constructions, circle language, Pythagoras and 2D trigonometry, mensuration and similarity; Higher adds the intersecting chord properties, the full circle-theorem reasoning programme, obtuse-angle and non-right-angled trigonometry, 3D problems, sectors, cones, spheres and the k^2/k^3 similarity ratios. On the June 2024 Higher papers this strand supplied the multi-step AO3 chains: a reasoned circle-theorem chase, a composite hexagon area, a 3D cosine-rule angle and a k^3 similar-solids equation.

Must-know for topic 4 (Higher formulae sheet gives the sine/cosine rules, $\frac{1}{2}ab \sin C$, and the cone/sphere volume and surface formulae):

Family	Formulae
triangles	$a^2 = b^2 + c^2 - 2bc \cos A$; $\frac{a}{\sin A} = \frac{b}{\sin B}$; area = $\frac{1}{2}ab \sin C$
circle	arc = $r\theta$ (θ in degrees: $\frac{\theta}{360} \times 2\pi r$); sector area = $\frac{1}{2}r^2\theta_{\text{rad}}$
solids	cylinder $V = \pi r^2 h$; cone $V = \frac{1}{3}\pi r^2 h$; sphere $V = \frac{4}{3}\pi r^3$, $SA = 4\pi r^2$
similarity	lengths k : areas k^2 : volumes k^3

Angles, polygons and symmetry

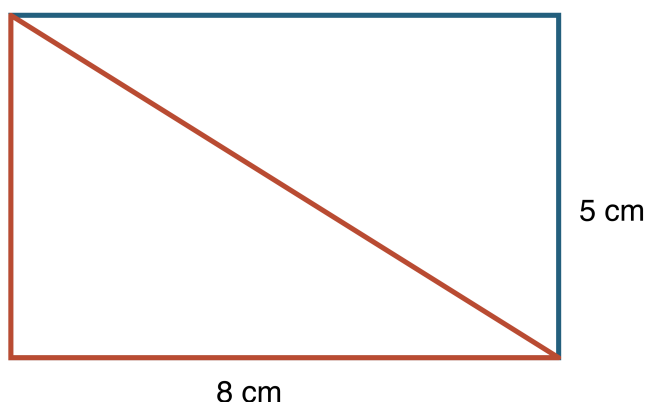
Angles classify as acute ($< 90^\circ$), right, obtuse (90° – 180°) and reflex ($> 180^\circ$). Angles on a straight line sum to 180° , around a point to 360° , vertically opposite angles are equal, and parallel lines give equal **corresponding angles** and **alternate angles** (allied angles sum to 180°). The exterior angle of a triangle equals the sum of the two remote interior angles, and the angle sum is 180° ; an isosceles triangle has two equal sides and base angles, an equilateral three of each at 60° .

Polygons carry names by side count; a **quadrilateral** has an angle sum of 360° , and each polygon's interior angles sum to $(n - 2) \times 180^\circ$. A **regular polygon** has all sides and angles equal: exterior angle = $360^\circ/n$, interior = $180^\circ - 360^\circ/n$ – a regular pentagon has interior angles of 108° . The special quadrilaterals nest: parallelogram (opposite sides and angles equal, diagonals bisect), rectangle (also right angles), rhombus (also all sides equal, diagonals perpendicular), square, kite, trapezium (one parallel pair). Two figures are **congruent** when identical in shape and size (conditions SSS, SAS, ASA, RHS); **similar** figures have equal angles with corresponding sides in ratio.

A shape's **line of symmetry** reflects it onto itself; its **order of rotational symmetry** is the number of self-matches in one turn (a rectangle: 2 lines, order 2; a regular hexagon: 6 lines, order 6).

Same base and height, half the area

Rectangle: 40 cm^2 ; triangle: 20 cm^2



Original native-lesson illustration; labels belong to its worked example.

Measures, bearings and compound rates

Read instrument scales sensibly and estimate before computing; time intervals work in both 12- and 24-hour clocks. An angle measure includes **three-figure bearings** : measured clockwise from north, written as three digits – 040° , not 40° . The return bearing reverses: if Q bears 040° from P, then P bears 220° from Q (add 180° , taking the answer mod 360).

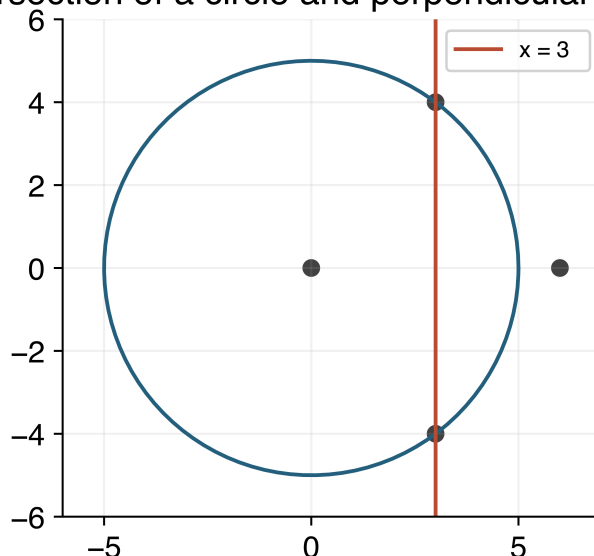
Average **speed** is distance over time; **density** is mass over volume; **pressure** is force over area – all one family of **compound measures** : fix the unit vocabulary (km/h , g/cm^3 , N/m^2) and convert carefully. A car covering 234 km in 2 h 36 min averages $234 \div 2.6 = 90 \text{ km/h}$.

Constructions and scale drawings

Measure and draw lines to the nearest millimetre; construct triangles from SSS, SAS or ASA with ruler and protractor. With **straight edge and compasses** construct (i) the perpendicular bisector of a line segment and (ii) the bisector of an angle; these also build specific angles (60° from an equilateral arc construction, 90° by perpendicular, 45° by bisecting). Constructions answer **locus** questions: the set of points a fixed distance from a point (circle), from a line (parallel pair), equidistant from two points (perpendicular bisector) or from two lines (angle bisector) – shade the region satisfying several conditions at once.

Scale drawings and maps carry a ratio (1 : 50 000): convert map distance to real distance through the scale factor, keeping units explicit.

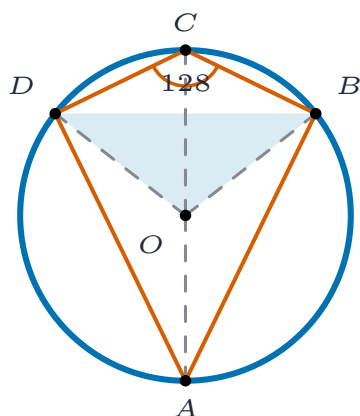
Intersection of a circle and perpendicular bisector



Original native-lesson illustration; labels belong to its worked example.

Circle geometry with reasons

Circle language: centre, radius, diameter, **chord**, circumference, **tangent** (touches at one point, perpendicular to the radius there), arc, sector, segment. Foundation uses chord and tangent properties informally; Higher requires the full programme with standard reasons. The **cyclic quadrilateral** has opposite angles summing to 180° ; the angle at the centre is twice the angle at the circumference on the same arc; angles in the same segment are equal; a diameter subtends a right angle (tangent–chord: alternate segment). Chord properties: the perpendicular from the centre bisects the chord; two intersecting chords satisfy $AE \times EB = CE \times ED$, and the internal/external versions power this family.



cyclic quad: $\angle BAD = 52$

centre = $2 \times$ rim: $\angle BOD = 104$

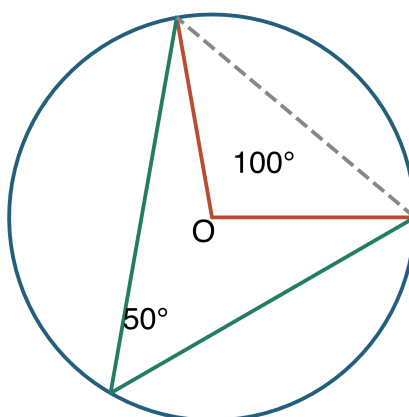
$OB = OD$ (radii): isosceles

$$\angle OBD = \frac{180 - 104}{2} = 38$$

every numerical step names its reason

Worked example (1HR June 2024 Q13, adapted). A, B, C, D lie on a circle centre O with $\angle BCD = 128^\circ$; find $\angle OBD$. Opposite angles of the cyclic quadrilateral sum to 180° , so $\angle BAD = 180 - 128 = 52^\circ$. The angle at the centre is twice the angle at the circumference on the same arc: $\angle BOD = 104^\circ$. Triangle OBD is isosceles ($OB = OD$, radii), so $\angle OBD = \frac{180 - 104}{2} = 38^\circ$. Every numerical step names its reason – that is what the marks list pays for.

The centre angle is twice the circumference angle



Original native-lesson illustration; labels belong to its worked example.

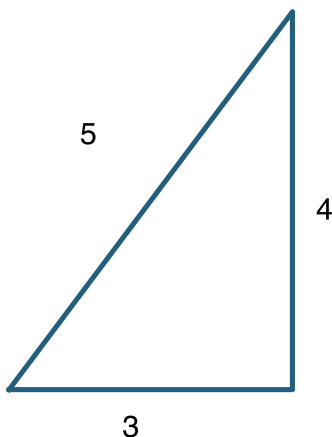
Pythagoras and trigonometry in 2D

Pythagoras' theorem : $a^2 + b^2 = c^2$ in right-angled triangles (6-8-10 is a whole-number triple; 5-12-13 another). SOH CAH TOA gives $\sin \theta = \frac{\text{opp}}{\text{hyp}}$, $\cos \theta = \frac{\text{adj}}{\text{hyp}}$, $\tan \theta = \frac{\text{opp}}{\text{adj}}$ – used forwards for sides and backwards (Shift-sin) for angles. The **angle of elevation** rises from the horizontal, the **angle of depression** falls from it (alternate angles carry it into the triangle).

Higher extends the ratios to **obtuse angles**: $\sin 130^\circ = \sin 50^\circ$ (sine stays positive), $\cos 130^\circ = -\cos 50^\circ$ (cosine turns negative). Any triangle now yields to the **sine rule**

$\frac{a}{\sin A} = \frac{b}{\sin B}$ (a pair – side over opposite angle – plus one more part) or the **cosine rule** $a^2 = b^2 + c^2 - 2bc \cos A$ (two sides and the included angle, or three sides), and the area formula $\frac{1}{2}ab \sin C$ works in reverse: area 15 with $a = 6$, $b = 10$ gives $\sin C = \frac{2 \times 15}{60} = 0.5$, so $C = 30^\circ$.

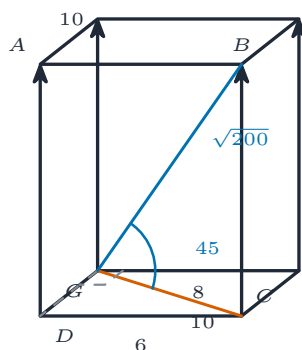
Label the known sides before choosing a rule



Original native-lesson illustration; labels belong to its worked example.

Trigonometry in three dimensions

3D problems are 2D problems inside a solid: name the right triangle, then Pythagoras or trigonometry as usual. In a cuboid with base 6 cm $\times$ 8 cm and height 10 cm, the base diagonal is $\sqrt{6^2 + 8^2} = 10$ cm, and the **space diagonal** is $\sqrt{6^2 + 8^2 + 10^2} = \sqrt{200} \approx 14.1$ cm; the angle between the space diagonal and the base diagonal satisfies $\tan \theta = \frac{10}{10}$, so $\theta = 45^\circ$.



$$\begin{aligned} \text{base } 6^2 + 8^2 &= 10^2; \text{ space} \\ 10^2 + 10^2 &= 200; \tan \theta = \frac{10}{10} \end{aligned}$$

Worked example (1HR June 2024 Q22, adapted). Points B , M , P in a solid box have $BM = \sqrt{200}$, $BP = \sqrt{244}$ and $MP = 10$ (each length itself found by Pythagoras in a right triangle of the box). Angle BMP needs the cosine rule on triangle BMP :

$$\cos BMP = \frac{200 + 100 - 244}{2 \times \sqrt{200} \times 10} = \frac{56}{282.8} = 0.198, \quad \angle BMP = 78.6^\circ$$

Order of attack: identify the target triangle, find its three sides (each possibly by its own Pythagoras), then apply the cosine or sine rule, keeping the angle you want at the centre of the notation.

Mensuration: perimeter, area, surface and volume

Metric conversions: linear $\text{cm} \leftrightarrow \text{m}$ by 100, area by 100^2 , **volume** by 100^3 ($1 \text{ m}^3 = 10^6 \text{ cm}^3$). Perimeters and areas build from triangles ($\frac{1}{2}bh$), rectangles, parallelograms (bh) and trapezia ($\frac{1}{2}(a+b)h$); the circle has circumference $2\pi r$ and area πr^2 (semicircles included). Composite shapes split into these parts.

Worked example (2HR June 2024 Q10, adapted). Hexagon $ABCDEF$ has $AB \parallel FC \parallel ED$ with $AB = 11$, $ED = FC = 23$, the perpendicular distance between FC and ED equal to 4.7 (cm) and the slant side $BC = 5$ making 30° with FC . The strip between FC and ED is a rectangle; above it the height of the trapezium $ABCF$ is $5 \sin 30^\circ = 2.5$. Split: rectangle $23 \times 4.7 = 108.1$ plus trapezium $\frac{1}{2}(11+23) \times 2.5 = 42.5$, so the area is 150.6 cm^2 (exact $\frac{753}{5}$). Draw the split lines before computing.

Higher adds **sectors** : arc $= \frac{\theta}{360} \times 2\pi r$, sector area $= \frac{\theta}{360} \times \pi r^2$ – for $r = 9$, $\theta = 40^\circ$: arc $= 2\pi \approx 6.28$, area $= 9\pi \approx 28.3$. Sector perimeter adds the two radii. Solids: prisms $V = \text{cross-section} \times \text{length}$ (cuboid, cylinder $V = \pi r^2 h$, surface $2\pi r^2 + 2\pi r h$); the **cone** $V = \frac{1}{3}\pi r^2 h$ with curved surface $\pi r l$ (l the slant); the **sphere** $V = \frac{4}{3}\pi r^3$, surface

$4\pi r^2$. A cone $r = 3$, $h = 4$ has $l = 5$ (Pythagoras), $V = 12\pi$, curved surface 15π .

Similarity: length, area and volume ratios

Similar figures have corresponding lengths in a fixed ratio k (the **scale factor**

): then areas scale by k^2 and volumes by k^3 – and conversely, an area ratio $4 : 9$ means a length ratio $2 : 3$. Maps and models use k directly.

Worked example (2HR June 2024 Q24 family, adapted). Solid B is similar to solid A with every length k times A 's. Volumes then scale by k^3 : the paper's cone-plus-hemisphere pair gives $k^3 = \frac{125}{8}$, so $k = \frac{5}{2} = 2.5$. A second pair with length ratio 4 has volume ratio $4^3 = 64$ – the larger solid holds 64 times the smaller. Set the ratio up on lengths first, then square or cube it once.

Checklist

- Classify angles; use line, point, parallel-line and triangle angle facts, including exterior-angle and polygon sums with regular-polygon interior/exterior angles (F 4.1, 4.2).
- Identify symmetry: lines and rotational order (F 4.3).
- Use bearings, speed/distance/time, density and pressure as compound measures (F 4.4).
- Construct triangles, bisectors and perpendiculars; answer locus and scale-drawing questions (F 4.5, F 4.11 B).
- Use circle language and chord/tangent properties; Higher: cyclic quadrilaterals, centre/circumference angles, intersecting chords – every step with its standard reason (F 4.6–4.7, H 4.6–4.7; June 2024 1HR Q13).
- Pythagoras and SOH CAH TOA in 2D with elevations and depressions; obtuse ratios, sine rule, cosine rule and $\frac{1}{2}ab \sin C$ including reversals (F 4.8, H 4.8 A–C, E).
- Solve 3D problems by extracting right triangles; cosine rule on the extracted triangle (H 4.8 D, F; June 2024 1HR Q22).
- Perimeters and areas including trapezia, circles and sectors; surface areas and volumes of prisms, cylinders, cones and spheres with unit conversions (F 4.9–4.10, H 4.9, H 4.10; June 2024 2HR Q10).
- Length : area : volume ratios $k : k^2 : k^3$, used both directions (F 4.11, H 4.11; June 2024 2HR Q24 family).

Boundary notes: bearings are three-figure; constructions are judged on visible arcs (leave them in); radians are excluded – degrees only; the ambiguous sine-rule case and 3D circle work are beyond the list. Foundation meets 4.1–4.11 Foundation rows only. Worked examples are adapted from the reviewed June 2024 Higher pair (see the meta's assessment evidence); sheet questions are original unless labelled.