

Sequences, functions and graphs

Pearson Edexcel · International GCSE · Mathematics A

4MA1 topic 3 (specification Issue 2, November 2017). Foundation rows 3.1 and 3.3 are shared with Higher; the function-notation block 3.2 and the calculus block 3.4 are Higher-only, and Higher 3.3 extends the graphs work to cubics, reciprocals, trig curves, transformations and intersections. On the June 2024 Higher papers this strand is the heaviest of the six topics (about a third of each paper) and carries the A/A* discriminators: algebraic series sums, composite functions, perpendicular lines, line–curve intersections and stationary points.

Must-know for topic 3 (Higher formulae sheet gives the series and trapezium rules only):

Family	Formulae
arithmetic	$u_n = a + (n - 1)d, \quad S_n = \frac{n}{2} [2a + (n - 1)d] = \frac{n}{2}(a + u_n)$
functions	$fg(x) = f(g(x))$ – do g first; f^{-1} undoes f on a one-to-one domain
lines	gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$; parallel $m_1 = m_2$; perpendicular $m_1 m_2 = -1$
intersections	$y_1 = y_2 \iff y_2 - y_1 = 0$ – solve, then substitute back
calculus	$\frac{d}{dx}(ax^n) = anx^{n-1}$; stationary where $\frac{dy}{dx} = 0$

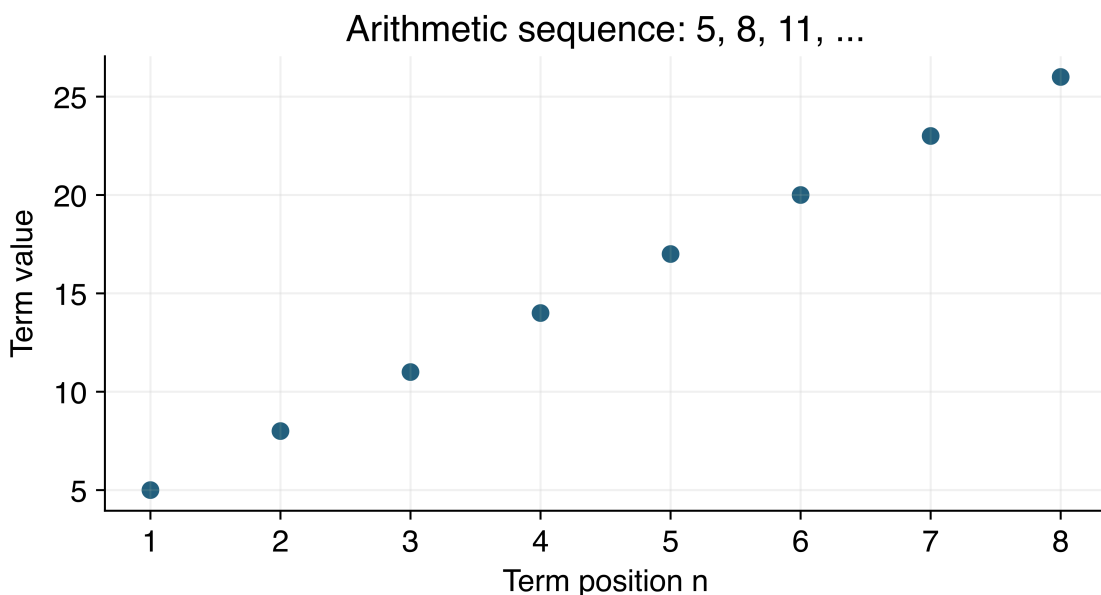
Sequences and series

A **term-to-term rule** says how to get the next term (add 4: 5, 9, 13, 17, ...; multiply by 2: 1, 2, 4, 8, ...); a **position-to-term rule** gives a term directly from its position. For an **arithmetic sequence** with first term a and **common difference** d , the n th term is $u_n = a + (n - 1)d$: for 1, 3, 5, 7, ... the rule is $2n - 1$, and "nth term is $4n + 3$ " generates 7, 11, 15, ... Check a proposed rule on two positions before trusting it.

Higher adds the **sum of an arithmetic series**. Pairing first with last, second with second-last, ... gives $n/2$ equal pair-sums of $a + u_n$:

$$S_n = \frac{n}{2} [2a + (n - 1)d] = \frac{n}{2}(a + u_n)$$

For $4 + 7 + 10 + 13 + \dots$: $a = 4$, $d = 3$, so $S_{50} = 25(8 + 49 \times 3) = 25 \times 155 = 3875$. Recovering a and d from two terms uses the term rule as simultaneous equations: if $u_2 = 7$ and $u_5 = 19$, then $3d = 12$, so $d = 4$ and $a = 7 - 4 = 3$.



Original native-lesson illustration; labels belong to its worked example.

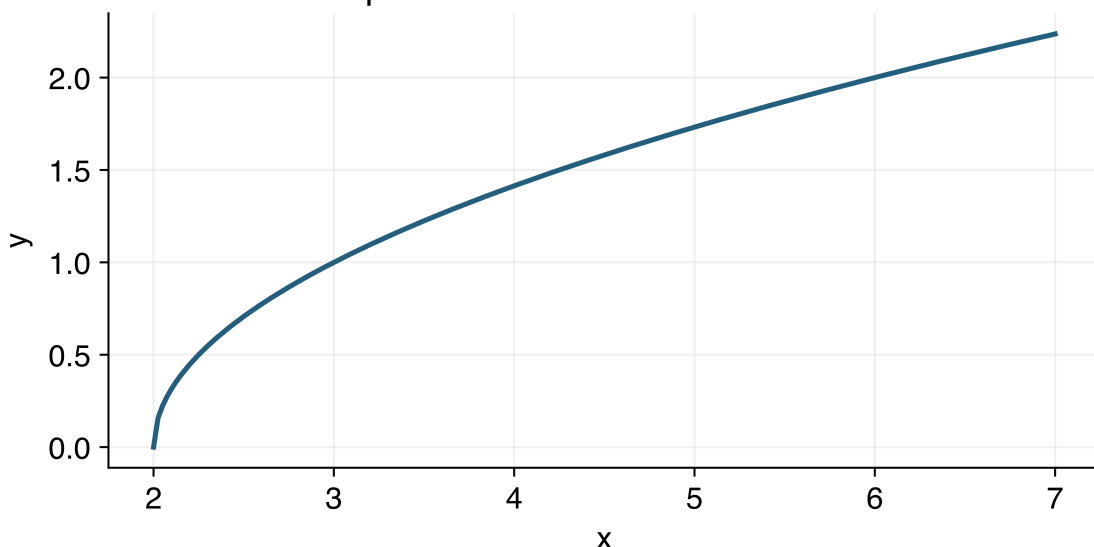
Worked example (1HR June 2024 Q23, adapted). Three consecutive terms of an arithmetic series are $4x - 14$, $x + 2$ and $7x - 9$. Consecutive terms carry a common difference, so $(x + 2) - (4x - 14) = (7x - 9) - (x + 2)$, giving $-3x + 16 = 6x - 11$, so $x = 3$. The terms are $-2, 5, 12, \dots$: $a = -2$, $d = 7$. Then $S_{40} = 20[2(-2) + 39 \times 7] = 20 \times 269 = 5380$. Keep a , d symbolic until the final substitution – the algebra, not the arithmetic, carries the marks.

Function notation

A **function** maps each input of one set to exactly one output of another: write $f(x) = \dots$ or $f: x \mapsto \dots$. The **domain** is the set of allowed inputs; the **range** is the set of outputs. Exclude inputs that break the rule: for $f(x) = \frac{1}{x-2}$ exclude $x = 2$; for $f(x) = \sqrt{x}$ take $x \geq 0$.

The **composite function** $fg(x) = f(g(x))$ applies g first, then f – the output of g must be an allowed input of f . The **inverse function** f^{-1} undoes f , and exists only where f is one-to-one: for $f(x) = \sqrt{x-2}$ (domain $x \geq 2$, range $f \geq 0$), writing $y = \sqrt{x-2}$ and squaring gives $x = y^2 + 2$, so $f^{-1}(x) = x^2 + 2$ on domain $x \geq 0$. Always check $f^{-1}(f(x)) = x$.

Square-root domain starts at $x = 2$



Original native-lesson illustration; labels belong to its worked example.

Worked example (2HR June 2024 Q17, adapted). $f(x) = \frac{x}{2x-4}$ and $g(x) = 3x+1$; find k given $fg(k) = 2$. First $g(k) = 3k+1$, then

$$fg(k) = f(3k+1) = \frac{3k+1}{2(3k+1)-4} = \frac{3k+1}{6k-2} = 2$$

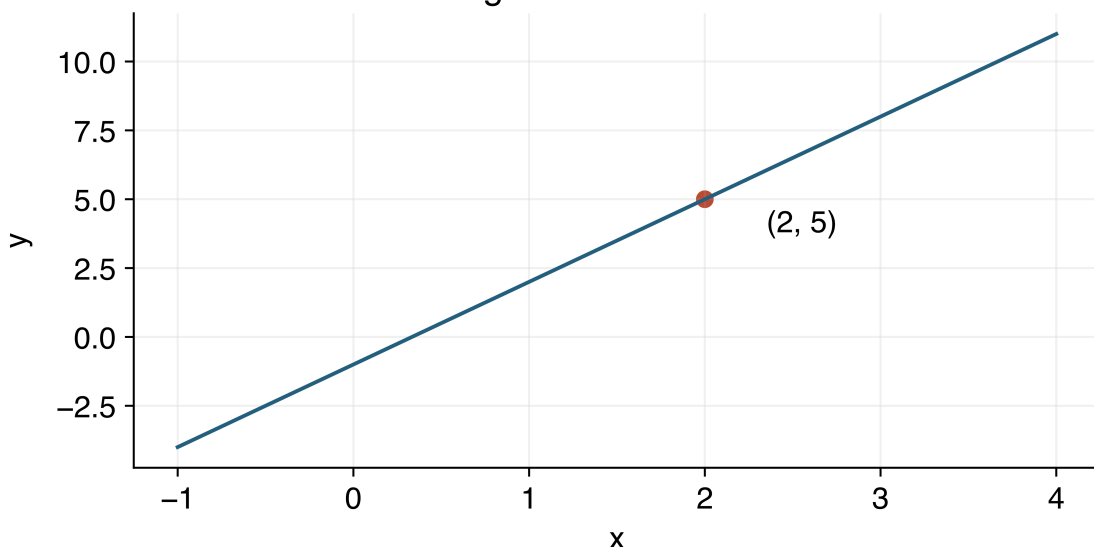
so $3k+1 = 12k-4$, giving $9k = 5$ and $k = \frac{5}{9}$. (Shortcut worth knowing: solving $f(x) = 2$ first gives $x = 8/3$, so set $g(k) = 8/3$ – same answer with less algebra.)

Coordinates and straight lines

Cartesian coordinates name points in all four quadrants; a **midpoint** averages the end coordinates: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. The **gradient** of a straight line is change in y divided by change in x : through $(1, 7)$ and $(2, 9)$ the gradient is 2. A line $y = mx + c$ has gradient m and crosses the y -axis at $(0, c)$ – for $y = 3x + 5$, gradient 3 and intercept $(0, 5)$. Special cases: $x = k$ is vertical (no finite gradient), $y = c$ is horizontal (gradient 0). Conversion graphs (currency, temperature) are read the same way.

Interpreting real graphs is part of this block: on a distance–time graph the gradient is speed; on a speed–time graph the gradient is acceleration and the area under the graph is distance travelled. Plotting practice covers filling tables of values for straight lines (including forms $ax + by = c$) and quadratics, drawing a smooth curve through plotted points – never line segments between them.

Point and gradient determine the line



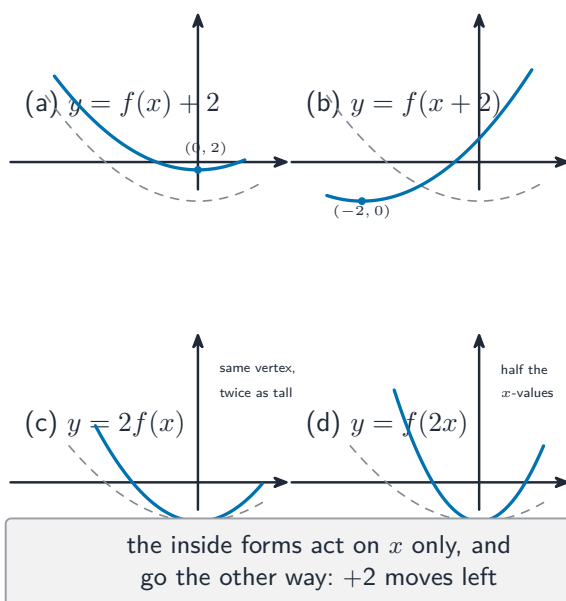
Original native-lesson illustration; labels belong to its worked example.

Parallel lines share a gradient. **Perpendicular lines** have gradients multiplying to -1 : perpendicular to m is $-1/m$. The line through (x_1, y_1) with gradient m is $y - y_1 = m(x - x_1)$.

Worked example (2HR June 2024 Q21, adapted). $ABCD$ is a square with $A(-5, 2)$ and $B(3, 5)$; find the equation of BC in the form $ax + by + c = 0$. Adjacent square sides are perpendicular. Gradient of AB is $\frac{5-2}{3-(-5)} = \frac{3}{8}$, so gradient of BC is $-\frac{8}{3}$. Through $B(3, 5)$: $y - 5 = -\frac{8}{3}(x - 3)$, so $3y - 15 = -8x + 24$, giving $8x + 3y - 39 = 0$. Multiply out at the end so a, b, c are integers, as asked.

Graphs and transformations

Higher 3.3 expects recognition and plotting of three curve families beyond quadratics: cubics $y = Ax^3 + Bx^2 + Cx + D$ (any signs; $y = x^3 - 4x$ rises–falls–rises), reciprocals $y = A/x$, $y = A/x^2$, $y = A/x^3$ (two branches, asymptotes, never crossing the axes) and the trigonometric graphs $y = \sin x$, $y = \cos x$, $y = \tan x$ for angles of any size in degrees – sine and cosine oscillate between -1 and 1 with period 360° ; tangent repeats every 180° and breaks at $90^\circ + 180^\circ n$. A circle centred at the origin has $x^2 + y^2 = r^2$ – the circle enters this course as the second curve of an intersection question.



Four **transformations** of $y = f(x)$ are required, for linear, quadratic, sine and cosine functions:

Form	Effect on $y = f(x)$
$y = f(x) + a$	translate by a vertically (up if $a > 0$)
$y = f(x + a)$	translate by $-a$ horizontally (left if $a > 0$)
$y = af(x)$	vertical stretch, scale factor a
$y = f(ax)$	horizontal stretch, scale factor $1/a$

The inside forms act "backwards" on x and only on x . For $f(x) = x^2$: $y = f(x-2)+5 = (x-2)^2 + 5$ moves the minimum from $(0, 0)$ to $(2, 5)$; $y = f(2x) = 4x^2$ halves the x -values, making the parabola narrower, not taller. For $y = \sin 2x$ the period halves to 180° . Read a drawn transformation off the graph (where did the turning point go?) and write it algebraically – both directions are assessed.

Gradients of curves: draw a tangent at the point and take its gradient as rise/run – the drawn line, not the curve, is measured. Exact curve gradients need calculus (next section).

Intersections: the x -values where a linear y_1 meets a non-linear y_2 are the solutions of $y_2 - y_1 = 0$. Meeting points of $y = 2x + 1$ and $y = x^2 + 3x - 2$ solve $x^2 + x - 3 = 0$; of $y = 5$ and $y = x^3 - 3x^2 + 7$, solve $x^3 - 3x^2 + 2 = 0$.

Worked example (2HR June 2024 Q22, adapted). Find where the line $y = 3x - 1$ meets the curve $x^2 + y^2 = y + 11$. Substitute the line into the curve:

$$x^2 + (3x - 1)^2 = (3x - 1) + 11 \Rightarrow 10x^2 - 9x - 9 = 0 \Rightarrow (5x + 3)(2x - 3) = 0$$

so $x = -\frac{3}{5}$ or $x = \frac{3}{2}$, with $y = 3x - 1$ giving $y = -\frac{14}{5}$ or $y = \frac{7}{2}$. The points are

$(-\frac{3}{5}, -\frac{14}{5})$ and $(\frac{3}{2}, \frac{7}{2})$ – substitute back into the *other* equation to check: $\frac{9}{4} + \frac{49}{4} = \frac{58}{4} = 14.5 = \frac{7}{2} + 11$. ✓

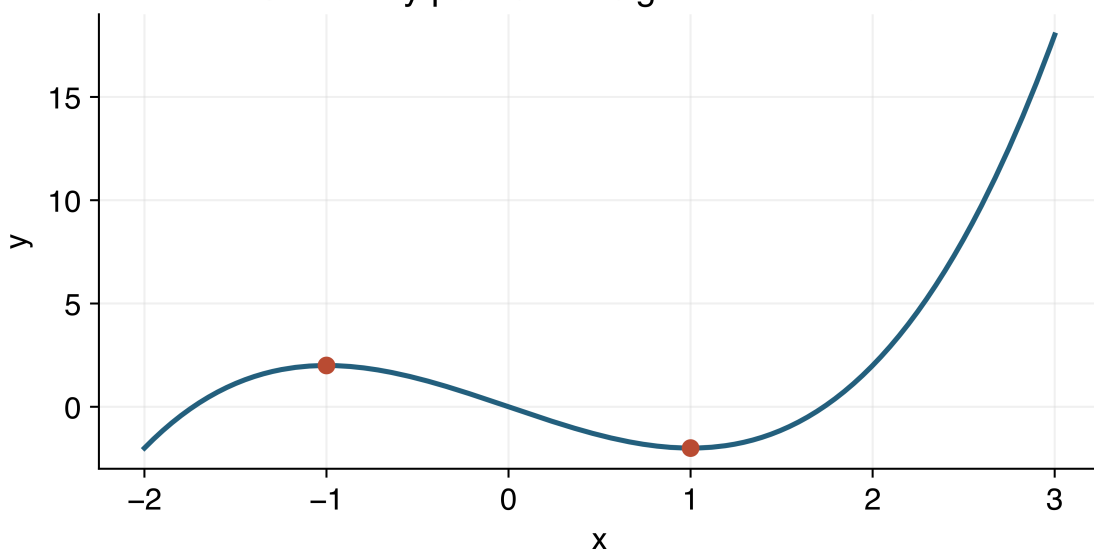
Differentiation and stationary points

The **gradient of a curve** varies along it: a **variable rate of change**.
Differentiation gives it exactly: for integer powers of x ,

$$y = ax^n \Rightarrow \frac{dy}{dx} = anx^{n-1}, \quad \text{term by term}$$

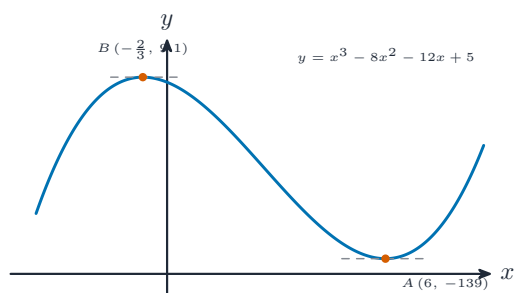
For $y = x^3 - 8x^2 - 12x + 5$: $\frac{dy}{dx} = 3x^2 - 16x - 12$. A **stationary point** has $\frac{dy}{dx} = 0$: here $3x^2 - 16x - 12 = (3x + 2)(x - 6)$, so $x = -\frac{2}{3}$ or $x = 6$. Distinguish **maximum** from **minimum** from the graph's shape: a cubic with positive x^3 coefficient rises–falls–rises... here $y \rightarrow -\infty$ as $x \rightarrow -\infty$, so the left stationary point $x = -\frac{2}{3}$ is the maximum ($y = \frac{247}{27} \approx 9.1$) and the right one is the minimum ($y(6) = 216 - 288 - 72 + 5 = -139$).

Stationary points and signed accumulation

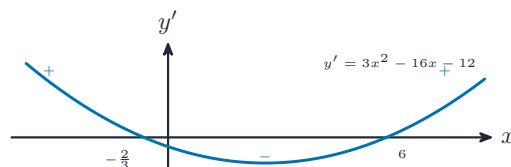


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Worked example (1HR June 2024 Q19, adapted). C has equation $y = x^3 - 8x^2 - 12x + 5$ and exactly two stationary points A and B with $x_A > x_B$; find A . Differentiate and set to zero: $(3x + 2)(x - 6) = 0$, and the larger root is $x = 6$. Substituting into the curve: $y = 216 - 288 - 72 + 5 = -139$, so $A = (6, -139)$. Marks follow the chain – differentiate (at least two terms right), set to zero, solve the quadratic, substitute back.



stationary where $\frac{dy}{dx} = 0$: $x = -\frac{2}{3}$ or 6



+ to - at B: maximum;
- to + at A: minimum

Kinematics: with displacement s in metres after t seconds, velocity is the rate of change of displacement and acceleration the rate of change of velocity: $v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$. For $s = t^3 - 24t$ (metres, $0 \leq t \leq 20$): $v = 3t^2 - 24$ m/s and $a = 6t$ m/s². The particle is instantaneously at rest where $v = 0$: $t = \sqrt{8} = 2.83$ s (3 s.f.).

Checklist

- Generate sequences from term-to-term and position-to-term rules; find the rule of an integer sequence (F 3.1 A–B).
- Use $u_n = a + (n - 1)d$ for arithmetic sequences; find a and d from any two terms (F 3.1 C, H 3.1 A–B).
- Sum an arithmetic series with $S_n = \frac{n}{2}[2a + (n - 1)d]$, including algebraic terms (H 3.1 C; June 2024 1HR Q23).
- Use $f(x)$ and $f: x \mapsto$ notation; state domains with exclusions and ranges (H 3.2 A–C).
- Build $fg(x) - g$ first – and find f^{-1} on a one-to-one domain (H 3.2 D; June 2024 2HR Q17).
- Read, plot and interpret graphs in all quadrants: midpoints, conversion graphs, distance/speed–time graphs, tables of values (F 3.3 A–I).
- Gradients from two points; $y = mx + c$; equations of parallel and perpendicular lines through a point (F 3.3 G–H, H 3.3 F–G; June 2024 2HR Q21).
- Recognise and sketch cubics, reciprocal curves, $\sin / \cos / \tan$, and the origin-centred circle $x^2 + y^2 = r^2$ (H 3.3 A).
- Apply and read off the four transformations $f(x) + a$, $f(x + a)$, $af(x)$, $f(ax)$ (H 3.3 B–C).
- Estimate a curve's gradient by drawing a tangent (H 3.3 D).
- Solve linear–non-linear intersections via $y_2 - y_1 = 0$ and check by substitution (H 3.3 E; June 2024 2HR Q22).

- Differentiate integer powers; find stationary points; classify max/min from the graph shape (H 3.4 A–D; June 2024 1HR Q19).
- Apply calculus to motion: $v = ds/dt$, $a = dv/dt$, and "at rest" means $v = 0$ (H 3.4 E).

Boundary notes: geometric series and sum-to-infinity are excluded at this level (no geometric rows in 4MA1 3.1); differentiation is limited to integer powers of x – chain, product and quotient rules are excluded; radian measure is excluded (degrees throughout). Foundation candidates meet 3.1 and 3.3 only. Worked examples above are adapted from the reviewed June 2024 Higher pair (see the meta's assessment evidence); all other questions on the sheets are original.