

Equations, formulae and identities

Pearson Edexcel · International GCSE · Mathematics A

4MA1 topic 2 (specification Issue 2, November 2017). Foundation rows 2.1–2.4, 2.6–2.8 are shared with Higher; proportion (2.5) is Higher-only, and Higher extends manipulation to fractional indices, triple expansions, algebraic fractions, completing the square, proofs, linear–quadratic pairs and quadratic inequalities. On the June 2024 Higher papers this strand again carries the A/A* discriminators: an index equation, recurring-decimal proof, algebraic-fraction division, completing the square with a transformed minimum, and a quadratic inequality.

Must-know for topic 2 (Higher formulae sheet gives the quadratic formula only):

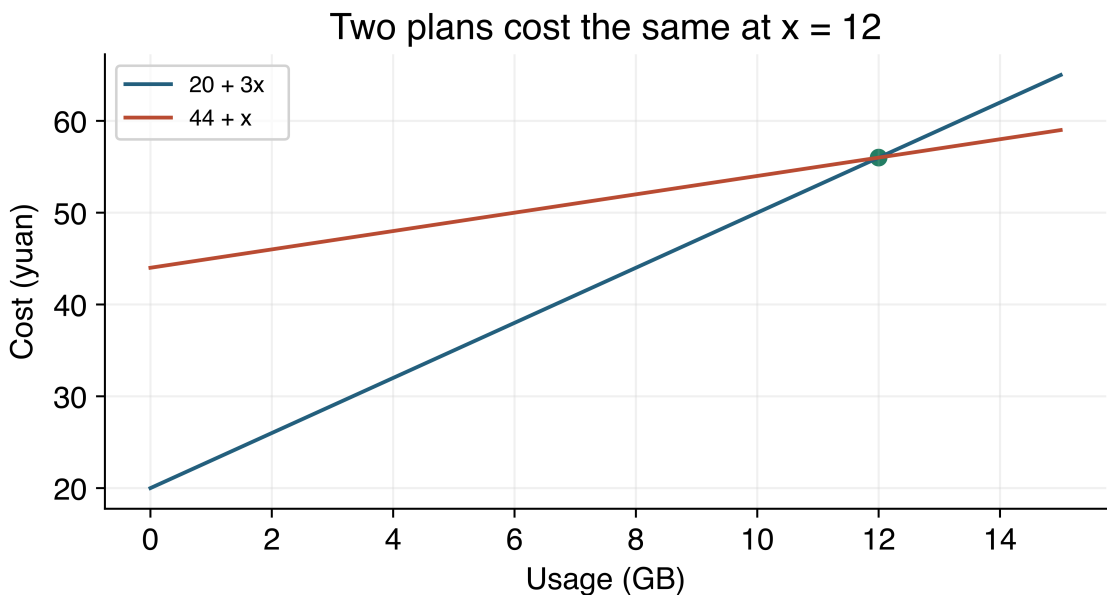
Family	Formulae
indices	$x^m \times x^n = x^{m+n}$, $x^m \div x^n = x^{m-n}$, $(x^m)^n = x^{mn}$, $x^0 = 1$, $x^{-n} = \frac{1}{x^n}$, $x^{1/2} = \sqrt{x}$
quadratics	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; factoring, or $a(x + b)^2 + c$ by completing the square
proportion	$y \propto x^n \iff y = kx^n$; inverse $y = k/x^n$

Symbols, indices and expressions

Symbols represent unknown numbers in equations or variables in expressions and formulae; algebraic expressions follow the generalised rules of arithmetic. An **equation** asks which inputs work; an **identity** holds for every allowed input. Index notation covers positive, negative and zero powers: $a^3 = a \times a \times a$, $a^0 = 1$, $a^{-1} = \frac{1}{a}$, and Higher adds fractional powers: $x^{1/2} = \sqrt{x}$, $x^{1/3} = \sqrt[3]{x}$, $x^{2/3} = (\sqrt[3]{x})^2$. The index laws combine them: $x^m \times x^n = x^{m+n}$, $x^m \div x^n = x^{m-n}$, $(x^m)^n = x^{mn}$ – multiply means add the indices; divide means subtract.

Worked example (2HR June 2024 Q7, adapted). $\frac{3^{n-2} \times 3^{n+5}}{3^{3n+10}} = 1$. Same base throughout, so the indices combine: $(n - 2) + (n + 5) - (3n + 10) = 0$, which is $-n - 7 = 0$. Hence $n = -7$. Never multiply the 3s – the bases match, so only the indices move.

Substitution evaluates expressions (negative and fractional values included); collecting like terms, multiplying a single term over a bracket, taking out common factors (factorise $x^2 + 10x + 24 = (x + 6)(x + 4)$) and expanding the product of two linear expressions ($((x + 8)(x - 5) = x^2 + 3x - 40)$ are the Foundation toolkit. Higher extends expansion to three brackets: $(x + 2)(x + 3)(x - 1) = (x^2 + 5x + 6)(x - 1) = x^3 + 4x^2 + x - 6$ – expand two first, then multiply through, and check with $x = 0$ (constant term -6).



Original native-lesson illustration; labels belong to its worked example.

Formulae and changing the subject

Use formulae from mathematics and other contexts, convert worded rules into symbols, and derive expressions from a situation. Changing the subject applies the same operation to both sides until the chosen letter stands alone: from $A = \pi r^2$, $r = \sqrt{A/\pi}$ (positive root – a length); from $v = u + at$, $t = \frac{v-u}{a}$.

Higher cases put the subject deeper in the formula. From $V = \frac{4}{3}\pi r^3$: multiply by 3, divide by 4π , then cube-root: $r = \sqrt[3]{\frac{3V}{4\pi}}$. From $T = 2\pi\sqrt{l/g}$: square first – $T^2 = \frac{4\pi^2 l}{g}$ – so $l = \frac{gT^2}{4\pi^2}$. When the subject appears twice, collect its terms before factorising: $4a + 3 = 5(a - r)$ expands to $4a + 3 = 5a - 5r$, so $a = 5r + 3$. Check every rearrangement by substituting numbers into both versions.

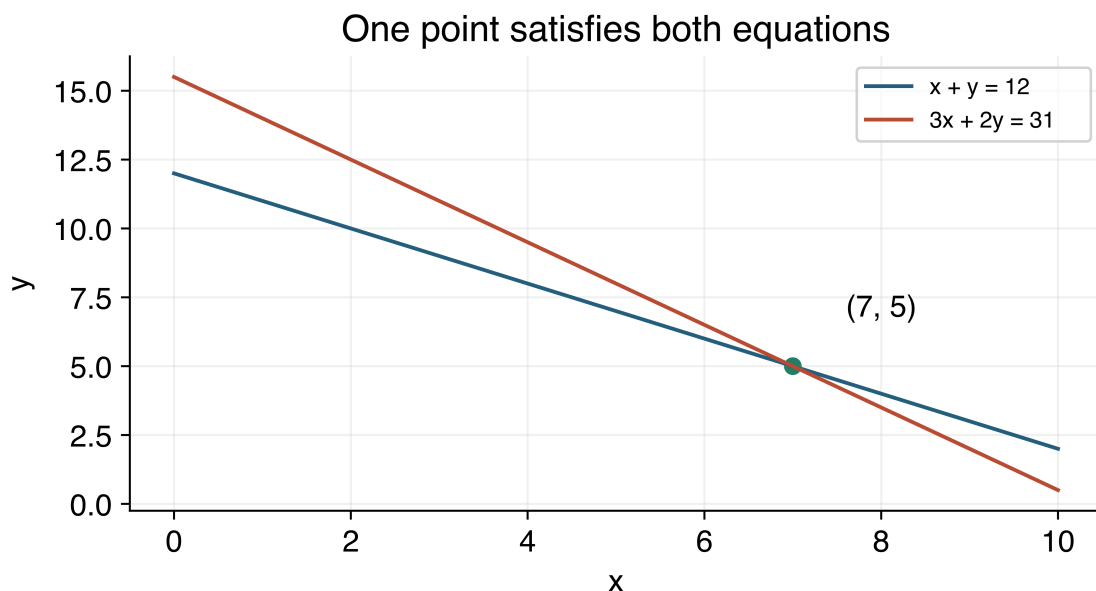
Proportion (Higher): $y \propto x^n$ means $y = kx^n$ for a constant k ; inverse proportion $y \propto 1/x^n$ means $y = k/x^n$. If $y \propto x^3$ and $y = 24$ when $x = 2$, then $k = 3$, so $y = 3x^3$ and $y = 81$ when $x = 3$. Graphically, $y = kx$ is a line through the origin; $y = k/x$ a hyperbola – the algebra and the graph must tell the same story.

Linear equations, proportion and simultaneous pairs

Solve linear equations with the unknown on either or both sides, integer or fractional coefficients – the moves are always expand, collect, divide: $7(x + 3) = 5x + 8$ gives $7x + 21 = 5x + 8$, so $2x = -13$ and $x = -6.5$; the fractional form $\frac{4}{5}x + 3 = \frac{2}{3}$ multiplies through by 15: $12x + 45 = 10$, so $x = -\frac{35}{12}$. Set up equations from given data: the three angles a° , $(a + 10)^\circ$, $(a + 20)^\circ$ of a triangle sum to 180, so $3a + 30 = 180$ and

$a = 50$. Check by substitution, and keep solutions sensible for the context (a count cannot be negative or fractional).

Two **simultaneous linear equations** solve exactly by elimination or substitution: for $2x + 3y = 17$ and $3x - 5y = 35$, triple the first and double the second: $6x + 9y = 51$, $6x - 10y = 70$; subtracting gives $19y = -19$, so $y = -1$ and $x = 10$. Check in the other equation: $3(10) - 5(-1) = 35$. ✓ Graphically each equation is a line and the common solution is their intersection point.



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Quadratic algebra

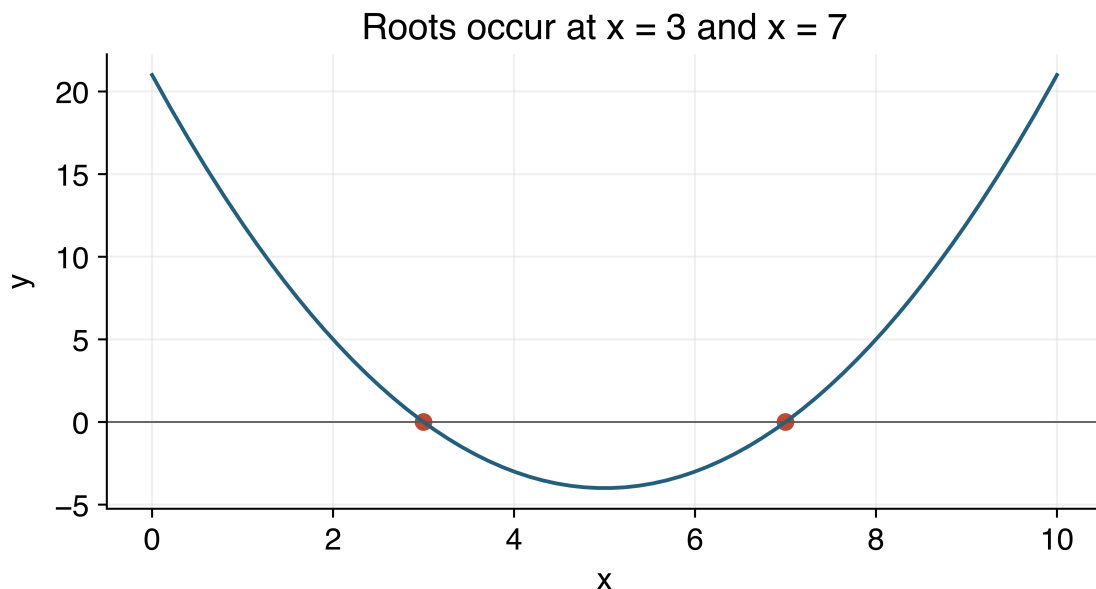
A **quadratic expression** has the form $ax^2 + bx + c$ with $a \neq 0$. Foundation factorises $x^2 + bx + c$; Higher factorises any: $6x^2 - 5x - 6 = (3x + 2)(2x - 3)$ – check by expanding ($6x^2 - 9x + 4x - 6$ ✓). Difference of two squares: $4x^2 - 9 = (2x - 3)(2x + 3)$.

Algebraic fractions : factor first, then cancel whole factors, never parts of terms. Simplify $\frac{x^2 + 3x - 10}{x^2 + x - 2} = \frac{(x + 5)(x - 2)}{(x + 2)(x - 1)}$ – no cancellation here (no shared factor); but $\frac{x^2 - 4}{x^2 + 5x + 6} = \frac{(x - 2)(x + 2)}{(x + 2)(x + 3)} = \frac{x - 2}{x + 3}$. Adding uses a common denominator; dividing means multiplying by the reciprocal.

Worked example (2HR June 2024 Q26, adapted). Write $4 - (3x - 5) \div \frac{3x^2 + x - 10}{4x - 1}$ as a single fraction. First factor the quadratic: $3x^2 + x - 10 = (3x - 5)(x + 2)$. Dividing by a fraction inverts it, and the $(3x - 5)$ cancels:

$$4 - \frac{(3x - 5)(4x - 1)}{(3x - 5)(x + 2)} = 4 - \frac{4x - 1}{x + 2} = \frac{4(x + 2) - (4x - 1)}{x + 2} = \frac{9}{x + 2}$$

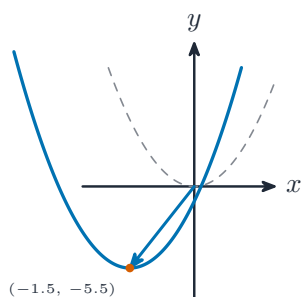
Proofs (Higher) use algebra, not examples. The classic recurring decimal: let $x = 0.306306\dots$; then $1000x = 306.306\dots$, and subtracting, $999x = 306$, so $x = \frac{306}{999} = \frac{34}{111}$. State what x equals, multiply by a power of ten that shifts one full recurring block, subtract, simplify (June 2024 2HR Q18).



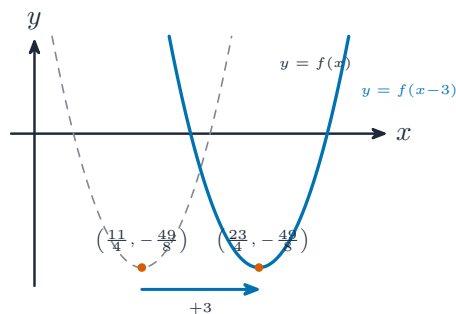
Original native-lesson illustration; labels belong to its worked example.

Completing the square and solving quadratics

Completing the square rewrites $ax^2 + bx + c = a(x + b')^2 + c'$: half the x -coefficient gives the shift, inside the bracket, with its sign flipped. The vertex form exposes the minimum (for $a > 0$) immediately: the graph of $y = a(x + b')^2 + c'$ is $y = ax^2$ translated by $(-b', c')$.



dashed $y = 2x^2$, solid $y = 2(x+1.5)^2 - 5.5$; the vertex moves to $(-1.5, -5.5)$



inside the bracket acts on x , the other way

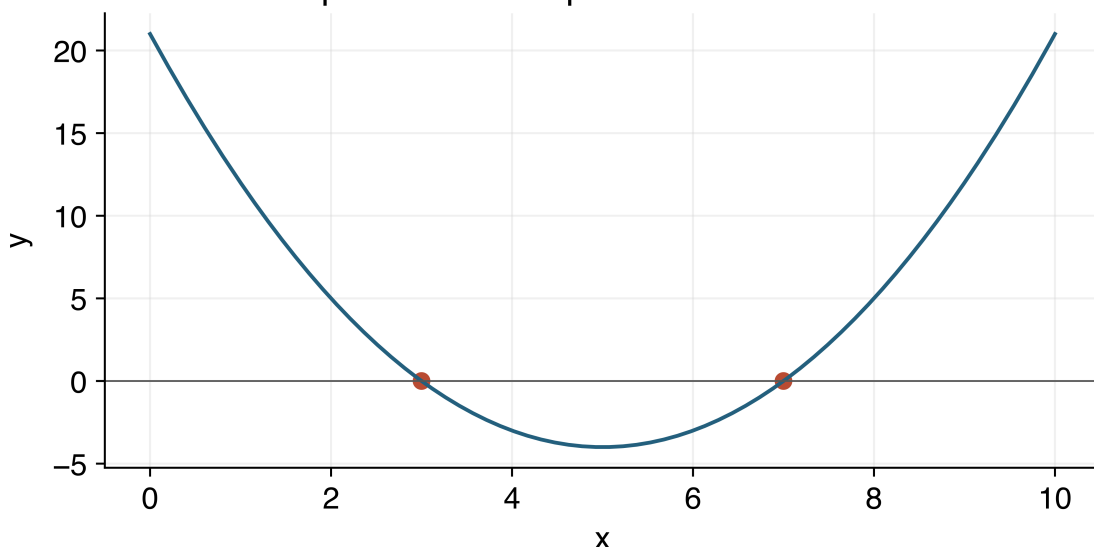
Worked example (1HR June 2024 Q20, adapted). (a) Write $2x^2 - 11x + 9$ in the form $a(x + b)^2 + c$: factor 2 from the x terms, half of $-\frac{11}{2}$ is $-\frac{11}{4}$, so

$$2x^2 - 11x + 9 = 2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + 9 = 2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$$

(b) The graph of $y = f(x - 3)$ shifts f three units right, so the minimum moves from $\left(\frac{11}{4}, -\frac{49}{8}\right)$ to $\left(\frac{23}{4}, -\frac{49}{8}\right) = (5.75, -6.125)$. Inside the bracket acts on x , and the other way.

Solving quadratics: by factorisation $(x^2 + x - 30 = (x+6)(x-5) = 0$ gives $x = -6$ or 5); by the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ when factorising fails – for $2x^2 + 6x - 1 = 0$, $x = \frac{-6 \pm \sqrt{36+8}}{4} = \frac{-3 \pm \sqrt{11}}{2}$; or by completing the square, which solves and gives the vertex at once. Form equations from context (area, Pythagoras, projectile-style $h = at^2 + bt + c$), keep exact answers unless told otherwise, and reject roots the context forbids.

The quadratic is nonpositive between 3 and 7

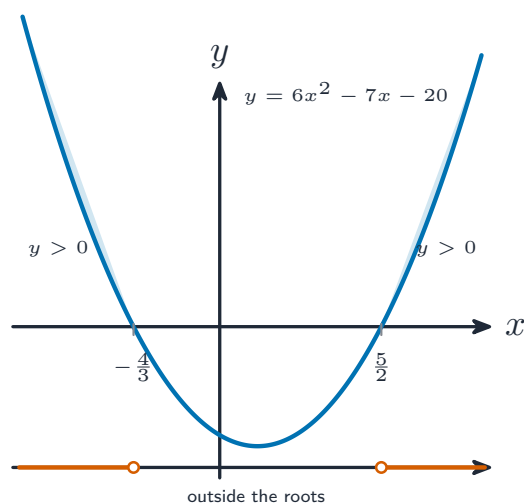


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Inequalities and linear–quadratic pairs

Inequality symbols $<$, $>$, $\leq$, $\geq$ (double-ended forms like $1 < x \leq 5$ included) describe intervals; open and closed circles on a number line mark excluded and included ends. Solving a linear inequality mirrors equation moves – except multiplying or dividing by a negative flips the direction: $7 - x \leq 5$ gives $-x \leq -2$, so $x \geq 2$.

Quadratic inequalities (Higher) need the critical values, then the sign of the parabola between them: a positive quadratic is above zero outside its roots.



positive quadratic: $>$ holds
outside the critical values

Worked example (2HR June 2024 Q20, adapted). Solve $6x^2 - 7x - 20 > 0$. Factorise: $(3x + 4)(2x - 5) > 0$, so the critical values are $x = -\frac{4}{3}$ and $x = \frac{5}{2}$. The parabola opens upward – positive outside the roots – so $x < -\frac{4}{3}$ or $x > \frac{5}{2}$, marked on the number line with open circles (the values themselves give 0, not > 0). Sketch the parabola first; the picture decides inside vs outside.

Linear–quadratic pairs (Higher): substitute the linear equation into the quadratic. For $y = 2x - 11$ with $x^2 + y^2 = 25$: $x^2 + (2x - 11)^2 = 25$ gives $5x^2 - 44x + 96 = 0$, and by formula $x = \frac{44 \pm \sqrt{1936 - 1920}}{10} = \frac{44 \pm 4}{10} = 4.8$ or 4 , giving $(4.8, -1.4)$ and $(4, -3)$; both satisfy the circle. For $y = 11x - 2$ with $y = 5x^2$: $5x^2 - 11x + 2 = (5x - 1)(x - 2) = 0$, so $(\frac{1}{5}, 0.2)$ and $(2, 20)$. Retain both solutions unless the context removes one.

Regions (F 2.8 D–E, harder cases Higher): on a Cartesian grid, each linear inequality bounds a half-plane; test one point per boundary (the origin is handy unless it lies on a boundary), then shade the region satisfying every condition – for example $x \geq 0$, $y \geq 1$, $x + y \leq 5$ gives the interior of a triangle with vertices $(0, 1)$, $(0, 5)$, $(4, 1)$.

Checklist

- Use symbols, index laws (integer, then fractional/negative/zero – Higher) and combine indices on one base (F 2.1, H 2.1; June 2024 2HR Q7).
- Substitute, collect terms, expand single and double brackets – and Higher, triple brackets – factorise common factors and quadratics (F 2.2, H 2.2 A–B).
- Derive, use and rearrange formulae, including subject-twice and power cases; set up and solve direct/inverse proportion (F 2.3, H 2.3, H 2.5).

- Solve linear equations from data and exact simultaneous pairs by elimination, interpreting the pair as intersecting lines (F 2.4, F/H 2.6).
- Manipulate algebraic fractions: factor, cancel, common denominators, invert to divide (H 2.2 C; June 2024 2HR Q26).
- Construct algebraic proofs, including recurring-decimal conversions (H 2.2 E; June 2024 2HR Q18).
- Complete the square, read the vertex form, and solve quadratics by factorising, formula or square – including from context (H 2.2 D, H 2.7 A–C; June 2024 1HR Q20).
- Solve linear and quadratic inequalities with number-line solution sets; identify bounded regions from linear inequalities (F 2.8, H 2.8; June 2024 2HR Q20).
- Solve linear–quadratic simultaneous pairs by substitution, keeping both points (H 2.7 D).

Boundary notes: the discriminant as a named tool and factor theorem are beyond the 4MA1 list – solving uses factorising, the formula or completing the square; surd rationalisation appears at Higher (rationalise $\frac{7}{4(2-\sqrt{3})}$ -style denominators by multiplying top and bottom by the conjugate, e.g. to $\frac{14+7\sqrt{3}}{4}$). Foundation meets 2.1–2.4, 2.6–2.8 only. Worked examples above are adapted from the reviewed June 2024 Higher pair (see the meta’s assessment evidence); sheet questions are original unless labelled.