

Numbers and the number system

Pearson Edexcel · International GCSE · Mathematics A

4MA1 topic 1 (specification Issue 2, November 2017) is the number heart of the qualification: Foundation rows 1.1–1.11 run from integers and the four rules through fractions, decimals, indices, sets, percentages, ratio, bounds and standard form to everyday number work; Higher adds recurring-decimal conversion, surds, algebraic sets, repeated percentage change and bound problems. On the June 2024 papers this strand supplied reverse percentage (2HR Q8), standard form (2HR Q9), the recurring-decimal proof (2HR Q18), upper-bound multiplication (2HR Q19), surds rationalisation (1HR Q17 family) and prime factors with Venn regions (1HR Q11 family).

Must-know for topic 1:

Family	Formulae
percentages	multiplier $1 + \frac{r}{100}$ (increase), $1 - \frac{r}{100}$ (decrease); reverse: original = final $\div$ multiplier
bounds	value v to 1 d.p. has LB = $v - 0.05$, UB = $v + 0.05$; maximise products by multiplying upper bounds
standard form	$a \times 10^n$ with $1 \leq a < 10$; indices add on multiplication, subtract on division
ratio	share by $\frac{\text{parts owned}}{\text{total parts}} \times \text{amount}$

Integers, four rules and the hierarchy of operations

Integers cover positive, negative and zero; place value names each digit's worth. Directed numbers appear in practical settings (temperature, debt, sea level); ordering puts them on the number line. The four rules with negatives: same signs add, different signs subtract, two negatives make a positive. **Brackets, then indices, then divide/multiply, then add/subtract** (BIDMAS): $3 + 4 \times 2^2 = 3 + 16 = 19$, while $(3+4) \times 2 = 14$. Odd, even, **prime numbers** (exactly two factors), **factors** and **multiples** classify whole numbers; common factors and common multiples lead to the HCF and LCM.

$$72 = 2^3 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

Shared smallest powers: $2 \times 3^2 = 18$

Original native-lesson illustration; labels belong to its worked example.

Fractions

Equivalent fractions cancel or extend: $\frac{8}{12} = \frac{2}{3}$. **Mixed numbers** and **vulgar fractions** convert by division with remainder. A **common denominator** lets fractions be ordered, compared, added and subtracted: $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$. A fraction OF a quantity multiplies: $\frac{3}{4}$ of 48 is 36; expressing one number as a fraction of another divides (7 as a fraction of 28 is $\frac{1}{4}$). Multiplying fractions multiplies numerators and denominators; dividing multiplies by the reciprocal – unit fractions as multiplicative inverses ($\frac{1}{3}$ undoes 3). Fractions convert to decimals (divide) and percentages (multiply by 100).

Recurring decimals (Higher) convert by algebra: let $x = 0.306306\dots$; then $1000x = 306.306\dots$; subtracting, $999x = 306$, so $x = \frac{306}{999} = \frac{34}{111}$ (June 2024 2HR Q18). A terminating decimal is a fraction with denominator a power of ten.

Decimals, indices, factors and multiples

Decimal place value runs tenths, hundredths, thousandths; ordering compares the highest-value digits first. **Square numbers** (1, 4, 9, ...), **cube numbers** (1, 8, 27, ...), their roots, and **index notation** with the multiplication and division laws for positive integer powers: $2^3 \times 2^4 = 2^7$ and $2^7 \div 2^3 = 2^4$. Every integer factors into primes: $360 = 2^3 \times 3^2 \times 5$; June 2024 1HR Q11 family factors large numbers into primes and reads products of regions from a Venn diagram ($19600 = 2^4 \times 5^2 \times 7^2$). The **HCF** takes the lowest power of each common prime; the **LCM** takes the highest power of every prime: for $360 = 2^3 \cdot 3^2 \cdot 5$ and $84 = 2^2 \cdot 3 \cdot 7$,

$\text{HCF} = 2^2 \cdot 3 = 12$ and $\text{LCM} = 2^3 \cdot 3^2 \cdot 5 \cdot 7 = 2520$.

$0.000072 = 7.2 \times 10^{-5}$

7.2 is the coefficient; -5 is the power

The coefficient is between 1 and 10

Original native-lesson illustration; labels belong to its worked example.

Surds

A **surd** is an exact irrational root such as $\sqrt{2}$. Simplify by extracting squares: $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$, using $\sqrt{ab} = \sqrt{a} \sqrt{b}$. Adding needs like surds: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$, but $\sqrt{2} + \sqrt{3}$ does not simplify.

Rationalising the denominator removes the root by multiplying top and bottom by the conjugate – a difference of two squares clears it exactly. Worked

case: $\frac{5}{3 - \sqrt{2}} = \frac{5(3 + \sqrt{2})}{(3 - \sqrt{2})(3 + \sqrt{2})} = \frac{5(3 + \sqrt{2})}{9 - 2} = \frac{5(3 + \sqrt{2})}{7}$; and $\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$. June 2024 1HR Q17 family rationalises a denominator built from $2 - \sqrt{3}$ to an answer of the family $\frac{7(2+\sqrt{3})}{4}$ – since $(2 - \sqrt{3})(2 + \sqrt{3}) = 1$, the $(2 + \sqrt{3})$ multiplier clears the surd exactly.

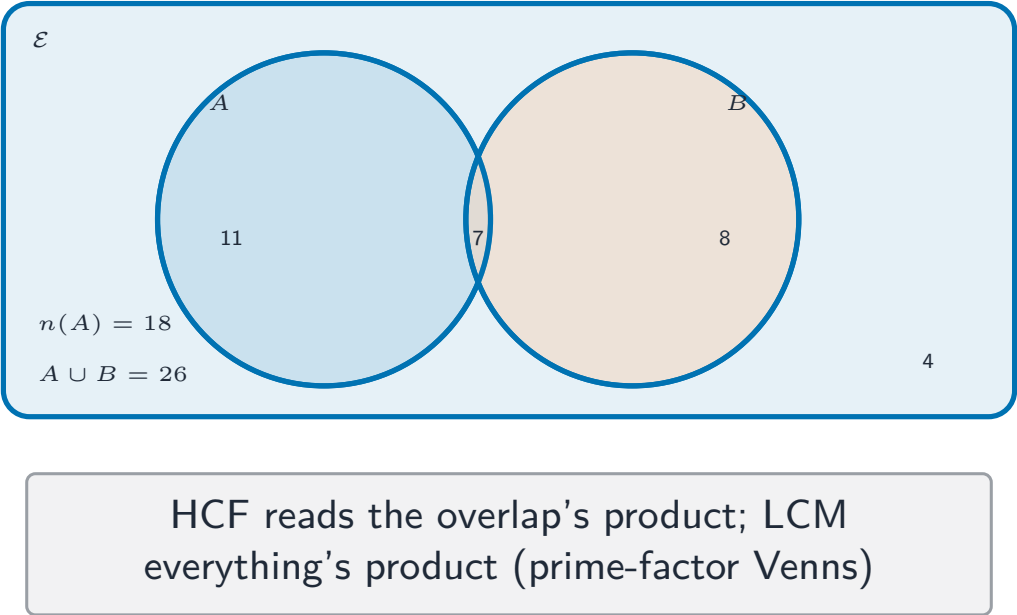
Sets

A **set** is a collection of elements: $A = \{2, 4, 6, 8\}$. Notation: $\cup$ (**union**, in either set), $\cap$ (**intersection**, in both), $\in$ and $\notin$ (**membership**).

The **universal set** $\mathcal{E}$ contains everything under discussion; the **empty set** $\emptyset$ has no elements; the **complement** A' holds everything in $\mathcal{E}$ outside A .

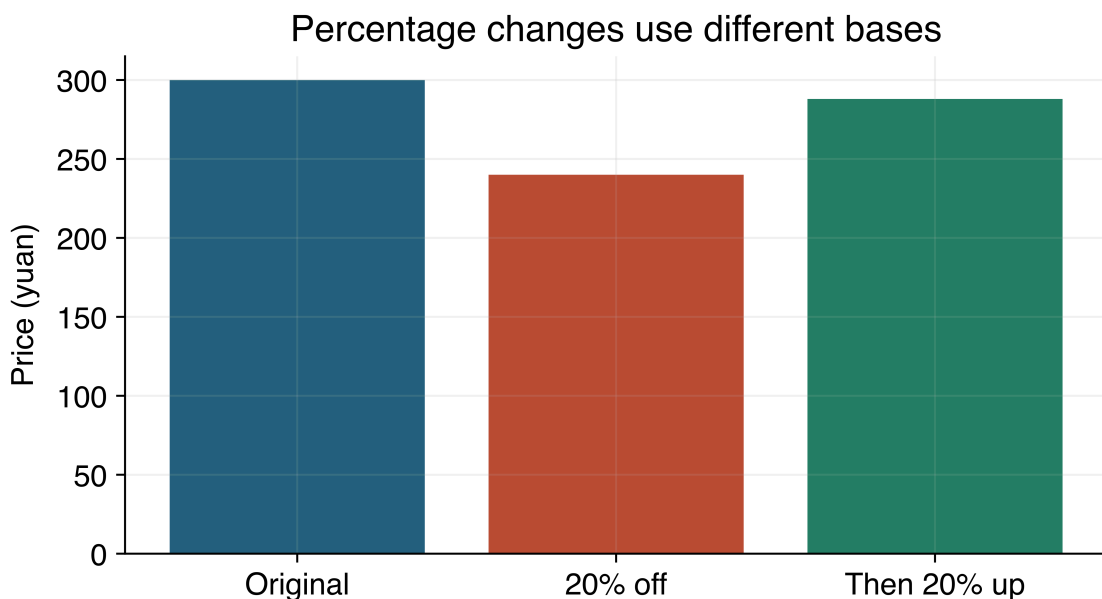
Venn diagrams draw the regions – at Higher with algebraic sets, **subsets** ($B \subset A$: every element of B lies in A), the element count $n(A)$, and practical uses (polls, prime-factor regions). June 2024 1HR Q11 family places the

prime factors of two numbers in a two-set Venn and reads products such as AB , A^2 and B^2 from combined regions – the HCF is the overlap’s product, the LCM everything’s product.



Percentages

Percentage means parts per 100: $35\% = 0.35 = \frac{7}{20}$. Percentages act as **multipliers**
: increase by 12% means $\times 1.12$; decrease by 20% means $\times 0.8$. Reverse percentage divides the final value by the multiplier – June 2024 2HR Q8: a fridge priced at 6225 rupees after a 17% reduction originally cost $\frac{6225}{0.83} = 7500$ rupees. **Compound interest** and depreciation apply the multiplier repeatedly: $A = P \left(1 \pm \frac{r}{100}\right)^n$ – £2000 at 3% compound for 4 years grows to $2000 \times 1.03^4 = \text{£}2251.02$; a car worth £12000 depreciating 15% per year for 3 years falls to $12000 \times 0.85^3 = \text{£}7369.50$.



Original native-lesson illustration; labels belong to its worked example.

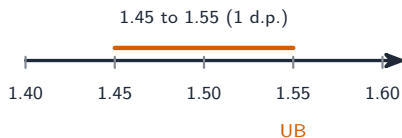
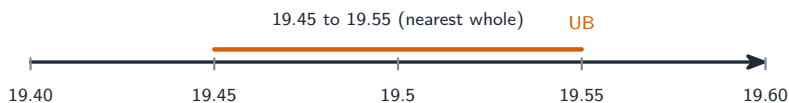
Ratio and proportion

Ratio compares parts: simplify by dividing through ($45 : 60 = 3 : 4$); share by $\frac{\text{parts}}{\text{total}} \times \text{amount}$ – £350 shared in the ratio $2 : 5$ gives $\frac{2}{7} \times 350 = \text{£}100$ and $\frac{5}{7} \times 350 = \text{£}250$.

Proportion scales within or between quantities: the unitary method finds one unit first (7 pens cost £4.20, so one pen costs £0.60 and 12 pens cost £7.20); quantities in **direct proportion** scale the same way ($y = kx$). Word problems mix ratio with fractions and percentages – keep the units explicit and check the answer is sensible.

Rounding, bounds, estimation and standard form

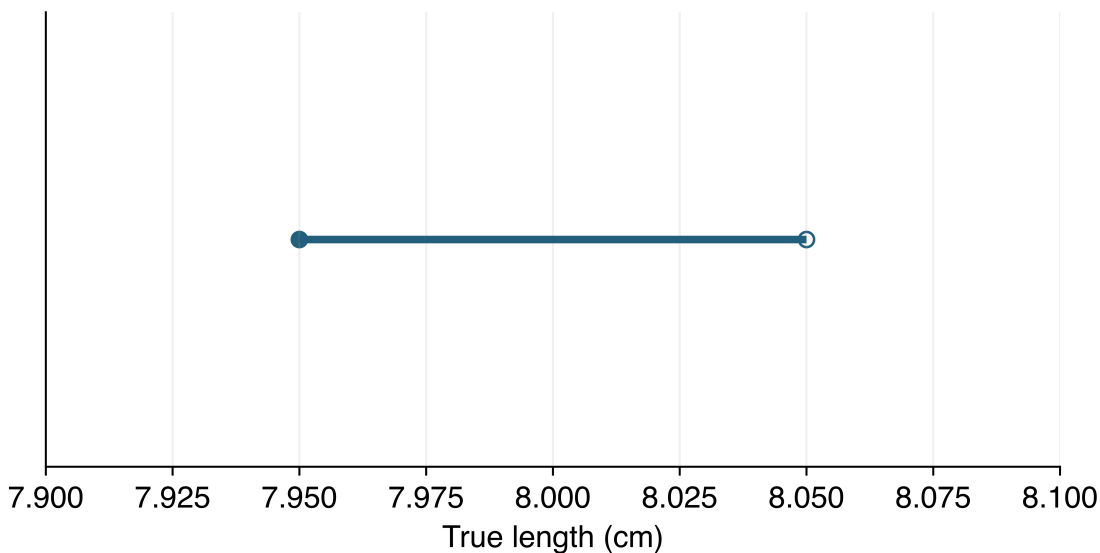
Round integers to powers of ten and decimals to significant figures or decimal places: $3.4617 \rightarrow 3.5$ (2 s.f.) and 3.46 (2 d.p.). An accurate value carries bounds – 19.5 to the nearest whole means $19.45 \leq v < 19.55$ – the **upper bound** 19.55 and **lower bound** 19.45. Estimation rounds every number to 1 s.f. first: $\frac{4.12 \times 29.8}{0.187} \approx \frac{4 \times 30}{0.2} = 600$. Bound problems (Higher) choose the extremes the question needs – June 2024 2HR Q19: distance = speed $\times$ time with speed 19 km/h to the nearest whole (so UB 19.5) and time 1.5 h to 1 d.p. (so UB 1.55): the upper bound of the distance is $19.5 \times 1.55 = 30.2$ km (3 s.f.).



maximise distance: UB speed $\times$ UB time
 $= 19.5 \times 1.55 = 30.2 \text{ km (3 s.f.)}$

Standard form writes $a \times 10^n$ with $1 \leq a < 10$: $604000 = 6.04 \times 10^5$ and $0.00007 = 7 \times 10^{-5}$ (June 2024 2HR Q9). Indices add and subtract across the multiplication and division of standard-form numbers, and the calculator's $\times 10^\wedge$ key keeps the working exact.

8.0 cm to nearest 0.1: $7.95 \leq L < 8.05$



Original native-lesson illustration; labels belong to its worked example.

Number in context: units, time and calculators

Everyday number – money, timetables, readings – uses the same arithmetic with units attached. Standard units: mass (kg, g), length (km, m, cm, mm), area, volume, time (seconds to years, 12- and 24-hour clocks), and the conversions between them ($1 \text{ m}^2 = 10^4 \text{ cm}^2$; $1 \text{ litre} = 1000 \text{ cm}^3$). A scientific calculator applies the order of operations correctly – but rounding early loses accuracy: keep full precision through the working and round only the final answer, to the accuracy asked for.

Checklist

- Integers, ordering, directed numbers, four rules, BIDMAS, odd, even, prime, factors and multiples (F 1.1 A–H).
- Equivalent fractions, mixed numbers, common denominators, fraction of a quantity, reciprocals, FDP conversions (F 1.2 A–H).
- Recurring decimals to fractions by algebra (H 1.3 A; June 2024 2HR Q18).
- Decimals: place value, ordering, terminating decimals as fractions (F 1.3 A–E).
- Squares, cubes, roots, index laws, prime factorisation, HCF and LCM (F 1.4 A–E; June 2024 1HR Q11 family).
- Surds: simplifying, adding like surds, rationalising with conjugates (H 1.4 A–B; June 2024 1HR Q17 family).
- Sets: notation, universal/empty/complement, Venn diagrams; Higher subsets, $n(A)$, algebraic and practical sets (F 1.5 A–E, H 1.5 A–D).
- Percentages: multipliers, percentage of, increase/decrease, reverse, compound interest and depreciation (F 1.6 A–G, H 1.6 A–B; June 2024 2HR Q8).
- Ratio in simplest form, sharing, direct proportion, word problems (F 1.7 A–E).
- Rounding to powers of ten, s.f. and d.p.; bound identification and bound problems choosing the right extremes; estimation by 1 s.f. (F 1.8 A–D, H 1.8 A; June 2024 2HR Q19).
- Standard form conversions and arithmetic (F 1.9 A, H 1.9 A; June 2024 2HR Q9).
- Everyday number, standard units and conversions, time calculations, calculator discipline (F 1.10 A–C, F 1.11 A).

Boundary notes: fractional, negative and zero indices beyond the multiplication and division laws belong to topic 2; proportion graphs belong to topics 2 and 4; compound measures (speed, density, pressure) belong to topic 4. Foundation candidates meet the Foundation rows only. Worked examples are adapted from the reviewed June 2024 papers or original; sheet questions are original unless labelled.